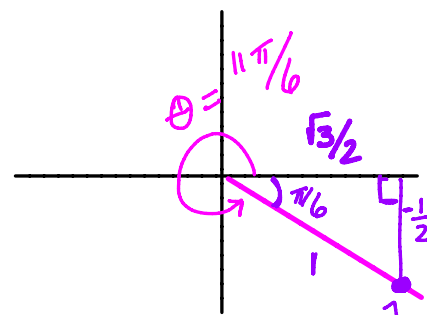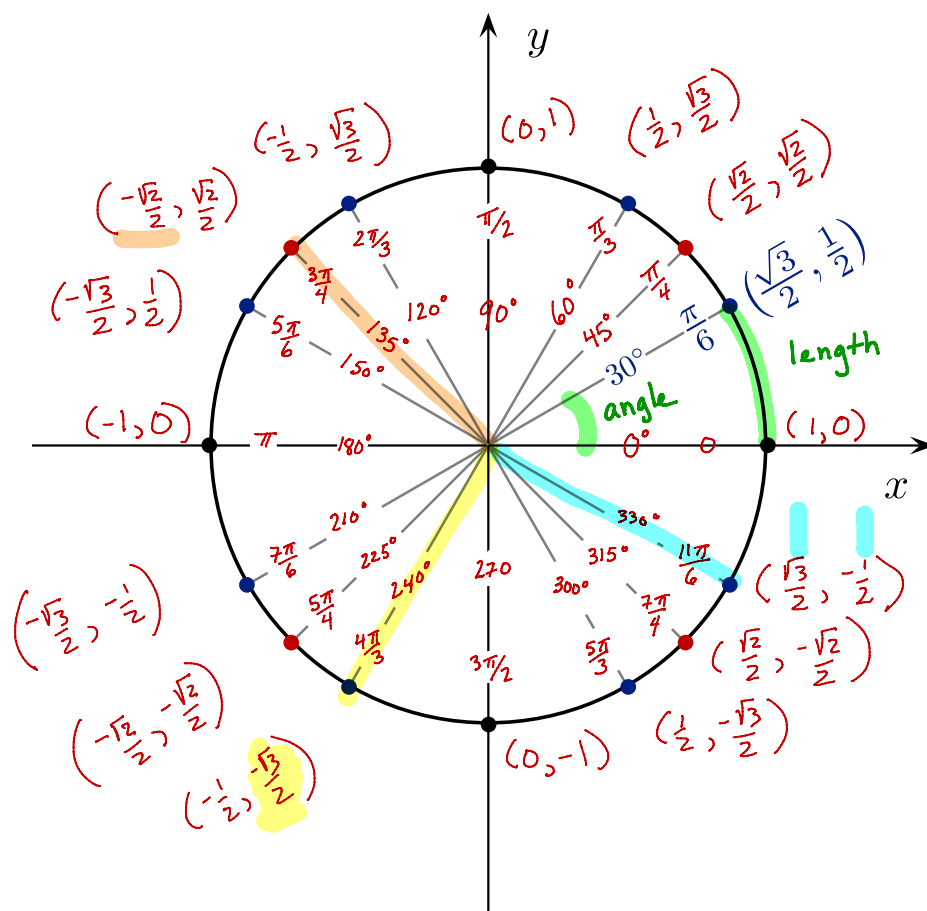1. **Unit Circle Definition**

(a) $\sin(4\pi/3) = \dfrac{-\sqrt{3}}{2}$

(b) $\cos(3\pi/4) = \dfrac{-\sqrt{2}}{2}$

(c) $\tan(11\pi/6) = \dfrac{\frac{-1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{-1}{\sqrt{3}}$

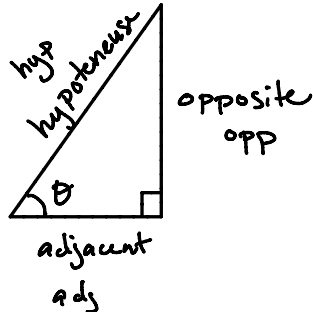$\dfrac{\sin\theta}{\cos\theta}$

What is a radian?   What is the circumference of the unit circle $(r=1)$?
$C = 2\pi r = 2\pi.$

So... compare length and angle for $\theta = 2\pi, \pi, \frac{\pi}{2}, \frac{\pi}{6} \ldots$

Recalculate

2. Right-triangle Definition

hyp hypotenuse

opposite opp

adjacent adj

θ

$\sin\theta = \dfrac{opp}{hyp}$

$\cos\theta = \dfrac{adj}{hyp}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{hyp}{opp}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{hyp}{adj}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\quad = \dfrac{opp}{adj}$

$\cot(\theta) = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

$\quad = \dfrac{adj}{opp}$

example

$\sqrt{2^2+4^2} = \sqrt{20} = 2\sqrt{5}$
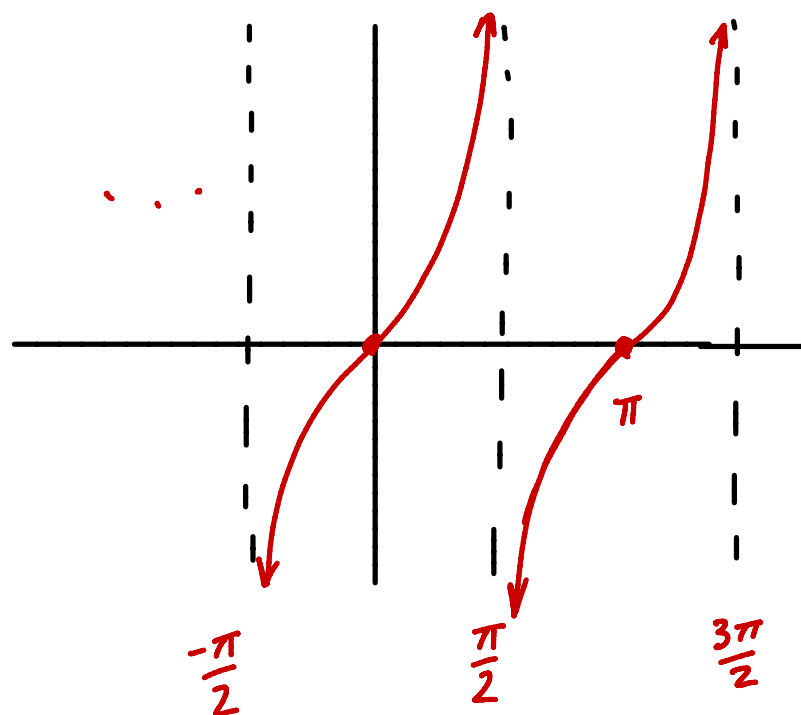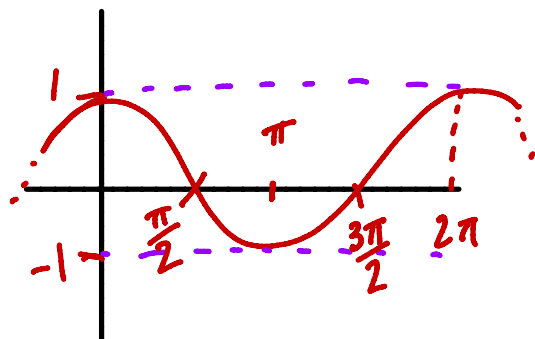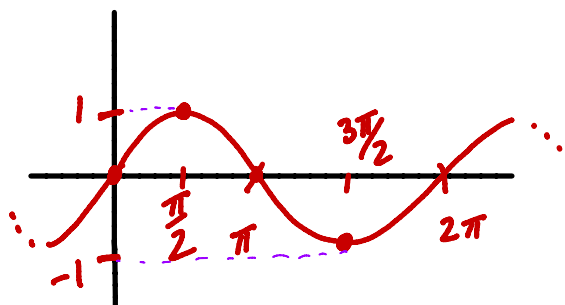
θ

4

2

Note:
No θ is even known!

$\sin\theta = \dfrac{2}{2\sqrt{5}} = \dfrac{1}{\sqrt{5}}$

$\cos\theta = \dfrac{4}{2\sqrt{5}} = \dfrac{2}{\sqrt{5}}$

$\tan\theta = \dfrac{2}{4} = \dfrac{1}{2}$

We can calculate everything on the previous page knowing relative lengths of <u>two</u> triangles:

1, 45°, $\dfrac{\sqrt{2}}{2}$, 45°, $\dfrac{\sqrt{3}}{2}$

1, 30°, $\dfrac{\sqrt{3}}{2}$, 60°, $\dfrac{1}{2}$
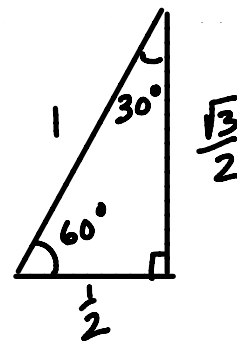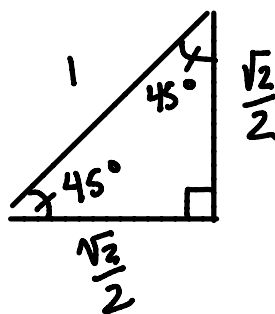
3. Familiar Graphs Use the previous work to construct and confirm the graphs of $f(\theta) = \sin(\theta)$, $f(\theta) = \cos(\theta)$, $f(\theta) = \tan(\theta)$.

4. Find *all* solutions to the equations below. Show your reasoning.

(c) $\tan x = 0$

(a) $\cos x = 1$

$X = ...-2\pi, 0, 2\pi, 4\pi, ...$

$X = ..., -\pi, 0, \pi, 2\pi, ...$

(b) $\sin x = 1$

$X = ... -2\pi + \frac{\pi}{2}, \frac{\pi}{2}, 2\pi + \frac{\pi}{2}, ...$

(d) $\sin x = 1/2$ (Find all solutions in $[0, 2\pi]$.)

$X = ..., -2\pi + \frac{\pi}{6}, \frac{\pi}{6}, 2\pi + \frac{\pi}{6}, 4\pi + \frac{\pi}{6}, ...$

and

$X = ..., -2\pi + \frac{5\pi}{6}, \frac{5\pi}{6}, 2\pi + \frac{5\pi}{6}, ...$

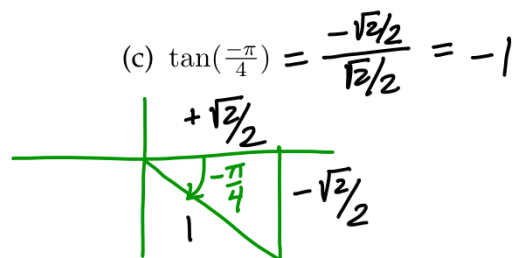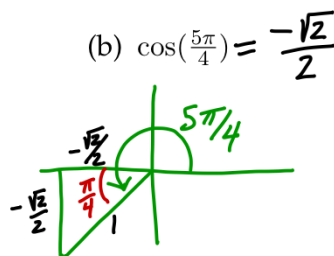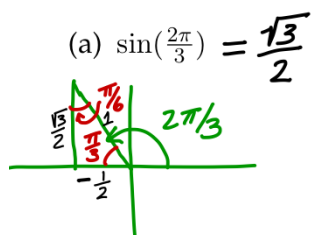5. Convert $2\pi/3$ radians and $5\pi/7$ radians to degrees.

$\frac{2\pi}{3} rad = 2\left(\frac{\pi}{3} rad\right) = 2 \cdot 60° = 120°$

$\left(\frac{5\pi}{7} rad\right)\left(\frac{180°}{\pi rad}\right) = \frac{5(180)}{7} = \frac{900}{7}$

6. Convert 20 degrees to radians.

$\left(20°\right)\left(\frac{\pi \, rad}{180°}\right) = \frac{20\pi}{180} = \frac{\pi}{9} rad$

7. Without a calculator and without going back to the first pages (!!) evaluate:

(a) $\sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$

(b) $\cos(\frac{5\pi}{4}) = \frac{-\sqrt{2}}{2}$

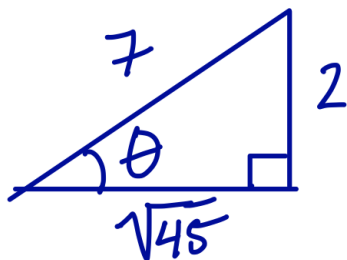(c) $\tan(\frac{-\pi}{4}) = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$

8. A wooden ramp is to be built with one end on the ground and the other end at the top of a short staircase. If the top of the staircase is 4 ft from the ground and the angle between the ground and the ramp is to be 10°, how long does the ramp need to be?

4 ft   h ← want h.
        $10°$
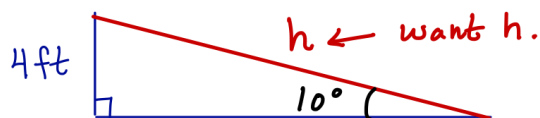
So $\sin(10°) = \frac{4}{h}$ or $h = \frac{4\,ft}{\sin(10°)} = 23ft$

make sure you use the correct units in your calculator!

9. Find $\cos\theta$ assuming that $\sin\theta = 2/7$ and θ is in the first quadrant.

7   2 $\sqrt{45}$
θ

$2^2 + x^2 = 7^2$

$x = \sqrt{49-4} = \sqrt{45}$

$\cos(\theta) = \sqrt{45}/7$