Your Name (Printed)　　　　　　　　　　　　　　End Time

## Directions

- You will have **one and a half** hours to complete the test. No extra time will be given, use your time wisely!

- This test is closed notes and closed book and you may **not** use a calculator.

- In order to receive full credit, you must **show your work** using correction notation. Please write out your computations on the exam paper. All answers should be simplified with the correct units where necessary.

- Simplify all answers by finding a common denominator, factoring out greatest common factors and canceling, when appropriate. In the exam you will be instructed to do this when the directions ask you to **simplify**.

- Solutions must be clearly identified by **placing a box around** your final answer to each question, when appropriate.

| Total Points | Your Score | Percent |
|:---:|:---:|:---:|
| 125 | | |

1. (10 points) Differentiate the following functions. **Do not simplify your answers.**

   (a) $y = 4x^5 + 8\sqrt[4]{x^5} + \sin 5$

   (b) $f(x) = \csc(2x) - \cot\left(\dfrac{x}{2}\right)$

2. (10 points) Find the derivatives of the following functions. **Simplify.**

   (a) $y = 2^{x \ln x}$

   (b) $h(x) = \dfrac{e^{1/x}}{x^2}$

3. (20 points) Find the derivatives of the following functions. **Simplify.**

(a) $y = \dfrac{x^2 - 4x + 2}{\sqrt{x}}$

(b) $y = \log_{10}(\sin(3x))$

(c) $f(x) = \tan\left(\dfrac{x}{1 + x^2}\right)$

(d) $y = \arctan(e^{7x^2})$

5. (15 points) Differentiate the following functions. **Simplify.**

(a) $y = x \arcsin(2x) + \frac{1}{2}\sqrt{1 - 4x^2}$

(b) $f(x) = \ln\left(\dfrac{(x^2 - 4)^5}{\sqrt{2x + 5}}\right)$

(c) $y = (\cos x)^x$

6. (6 points) Find the equation of the tangent line to $f(x) = \sqrt{1 + 4\sin x}$ at the point $(0, 1)$. Give your answer in slope-intercept form.

7. (6 points) Find all values of $x$ for which the graph of $f(x) = \sin x + \cos^2 x$ has a horizontal tangent line. Note, you only have to find the $x$-coordinate.

8. (5 points) Find the 46th derivative of $g(x) = e^{5x}$.

9. (6 points) Given $xe^y = y - 1$ find $\dfrac{dy}{dx}$.

10. (6 points) Find the equation of the tangent line to $x^2 + 4xy + y^2 = 13$ at the point $(2, 1)$. Give your answer in slope-intercept form.

11. (7 points) Each side of a square is increasing at a rate of 6 cm/sec. At what rate is the area of the square increasing when the area of the square is 16 cm$^2$? Give your answer with proper units.

12. (7 points) A boat is pulled into a dock by a rope attached to the bow (front) of the boat and passing through a pulley on the dock that is 1 m higher than the boat. If the rope is pulled in at a rate of 2 m/sec, how fast is the boat approaching the dock when it is 3 m from the dock? Give your answer with proper units.

13. (7 points) A water tank has the shape of a cone (point down) with height 5 m and base radius 2 m. Suppose the tank is full initially, and water is leaking from the tank at a rate of 0.5 m$^3$/min. How fast is the water level falling when the water is 3 m deep? ($V = \frac{1}{3}\pi r^2 h$)

14. (6 points) Find the linearization of $f(x) = e^x$ at $a = 0$ and use it to estimate $e^{0.1}$

15. (8 points) A particle moves along a horizontal line so that its coordinate at time $t$ is $x = t^3 - 3t + 5, t \geq 0$.

    (a) (2 points) Find the velocity and acceleration functions.

    (b) (2 points) When is the particle at rest?

    (c) (4 points) Draw a diagram of the particle's position and find the distance the particle travels in first three seconds.

16. (6 points) Use differentials to estimate the amount of paint needed to apply a coat of paint 0.1 **cm** thick to a hemispherical dome with radius 10 **m**. Give your final answer with proper units. (The volume of a sphere is $V = \frac{4}{3}\pi r^3$)