Your Name

Solutions

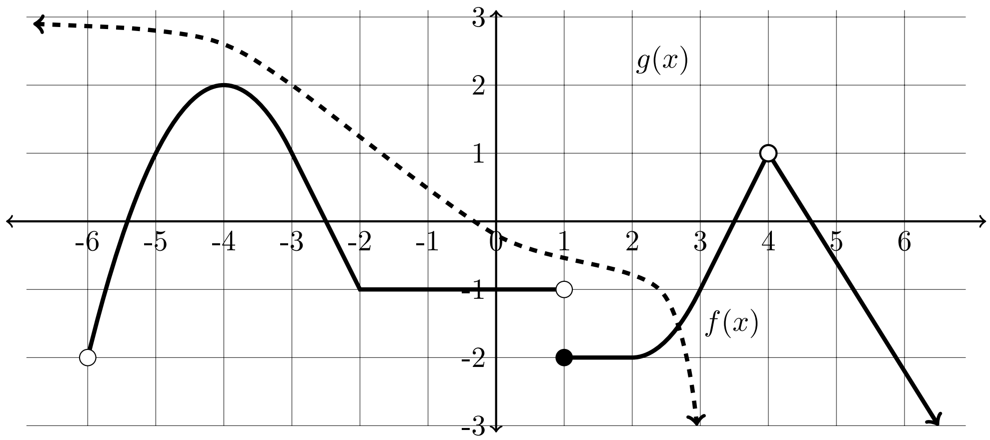
Your Signature

Instructor Name

Problem	Total Points	Score
1	10	
2	12	
3	10	
4	10	
5	10	
6	8	
7	20	
8	10	
Extra Credit	(5)	
Total	100	

- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work.** Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- Raise your hand if you have a question.

1. (10 points) The graphs of two functions $f(x)$, shown thick, and $g(x)$, shown dashed, are given below. Determine the following. If the value does not exist, write "DNE".



(a) $\lim_{x \to -6^+} f(x) =$ <u>-2</u>

(b) $\lim_{x \to -3} f(x) =$ <u>1</u>

(c) $\lim_{x \to \infty} f(x) =$ <u>$-\infty$</u>

(d) $\lim_{x \to 1} f(x) =$ <u>DNE</u>

(e) $f'(-4) =$ <u>0</u>

(f) $\lim_{x \to 3^-} g(x) =$ <u>$-\infty$</u>

(g) $\lim_{x \to -\infty} g(x) =$ <u>3</u>

(h) $\lim_{x \to -3} f(g(x)) =$ <u>-2</u>

(i) What is the domain of $f(x)$? Give your answer in interval form.

$(-6,1) \cup (1,\infty)$

(j) Where in the domain of $f(x)$ is the function continuous? Give your answer in interval form.

$(-6,1) \cup (1,4) \cup (4,\infty)$ or

everywhere except when $x=4$

2. (12 points) Consider the function

$$g(x) = 5 + 7x^2.$$

Using the **definition of the derivative**, find $g'(a)$. No credit will be given if a different method is used. [It is recommended you start by writing the definition of the derivative.]

$$g'(a) = \lim_{h \to 0} \frac{g(a+h) - g(a)}{h}$$

$$= \lim_{h \to 0} \frac{5 + 7(a+h)^2 - (5 + 7a^2)}{h}$$

$$= \lim_{h \to 0} \frac{5 + 7a^2 + 14ah + 7h^2 - 5 - 7a^2}{h}$$

$$= \lim_{h \to 0} \frac{14ah + 7h^2}{h} = \lim_{h \to 0} \frac{h(14a + 7h)}{h}$$

$$= \lim_{h \to 0} 14a + 7h = 14a.$$

Summary: $f'(a) = 14a$.

3. (10 points) Let f be the piecewise defined function below.

$$f(x) = \begin{cases} c - \cos(x) & x \neq \pi \\ 4 & x = \pi \end{cases}$$

(a) Determine a value for c such that the function $f(x)$ is continuous at $x = \pi$.

Pick $c = 3$

(b) Show that your choice for c above is correct using the *definition of continuity at a point.* (A correct answer will involve writing and computing an appropriate limit or limits.)

$f(\pi) = 4$,

$\lim_{x \to \pi} f(x) = \lim_{x \to \pi} 3 - \cos x = 3 - \cos(\pi) = 3 + 1 = 4$

Since $f(\pi) = 4 = \lim_{x \to \pi} f(x)$, $f(x)$ is continuous at $x = \pi$.

4. (10 points) The function $f(x) = 3 + 5x + e^{2x}$ has derivative $f'(x) = 5 + 2e^{2x}$.

(a) Find $f(0)$.

$f(0) = 3 + 0 + e^0 = 4$

$f(0) =$ 4

(b) Find $f'(0)$.

$f'(0) = 5 + 2 \cdot e^0 = 7$

$f'(0) =$ 7

(c) Write an equation of the line tangent to the graph of $f(x)$ when $x = 0$ in slope-intercept form. (That is, in the form $y = mx + b$.)

$y - 4 = 7(x - 0)$

$y = 7x + 4$

Equation of Tangent Line: $y = 7x + 4$

5. (10 points) Let $g(x) = \frac{1}{x-3} + 2$.

(a) Evaluate the limit: $\lim_{x\to 3^-} f(x)$. Show your work or justify your computation with a few words.

$\lim_{x\to 3^-} \left(\frac{1}{x-3} + 2\right) = -\infty$ since

(like $x = 2.99$) as $x \to 3^-$, $x-3 \to 0^-$ and $1>0$

$\lim_{x\to 3^-} f(x) =$ $-\infty$

(b) What does your answer to part (a) tell you about the graph of $f(x)$? (What feature does the graph have?)

a vertical asymptote at $x=3$

(c) Evaluate the limit: $\lim_{x\to -\infty} f(x)$. Show your work or justify your computation with a few words.

$\lim_{x\to -\infty} \left(\frac{1}{x-3} + 2\right) = 2$

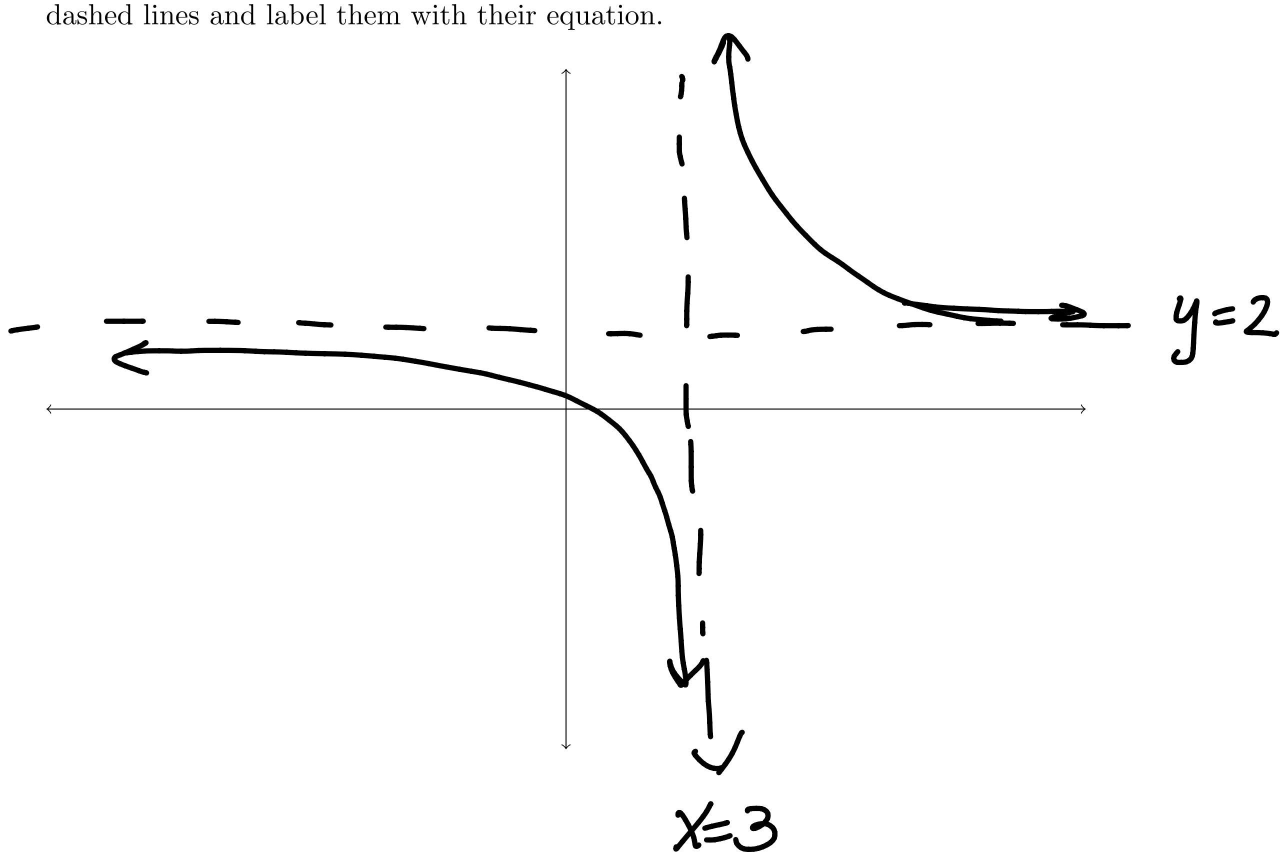
since $\frac{1}{x-3} \to 0$ as $x \to -\infty$

$\lim_{x\to -\infty} f(x) =$ 2

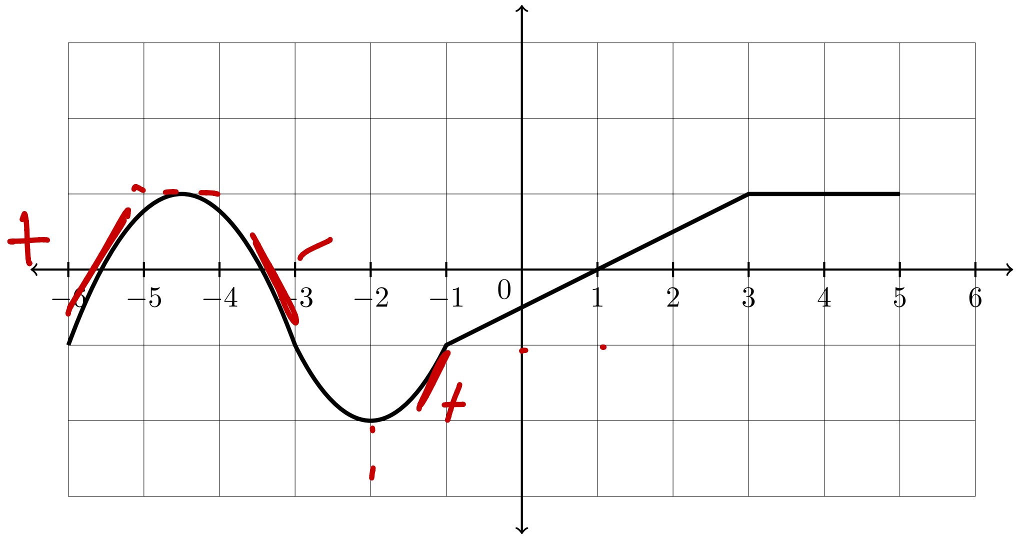
(d) What does your answer to part (c) tell you about the graph of $f(x)$? (What feature does the graph have?)

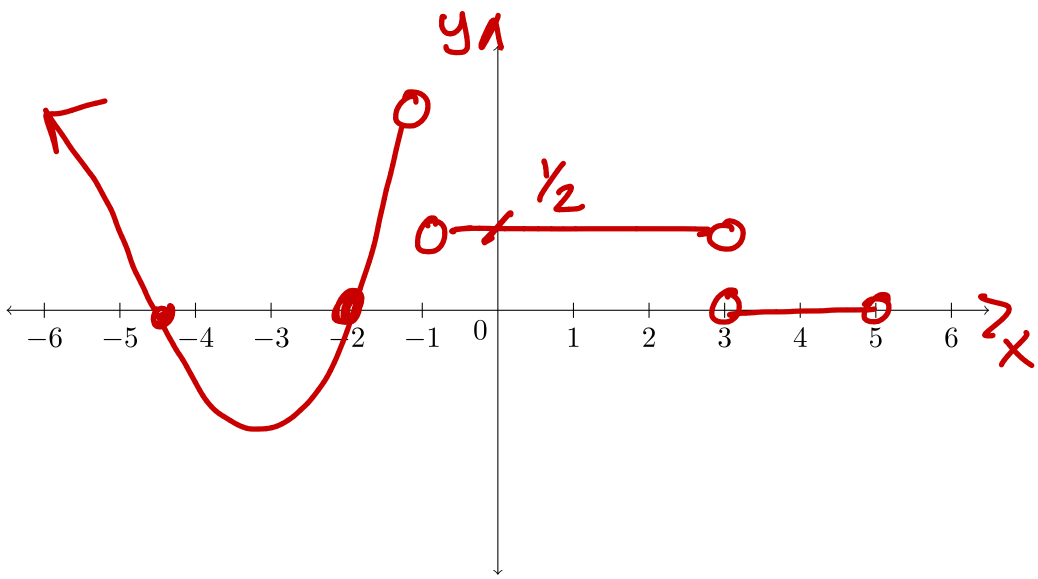
a horizontal asymptote at $y=2$

(e) Sketch the graph of $f(x)$ on the axes below. If the graph has any asymptotes, draw them as dashed lines and label them with their equation.



6. (8 points) Sketch the derivative of the following function, on the second set of axes.





7. (20 points) Find the limit or show it doesn't exist. Use proper limit notation for full credit. If a limit is positive or negative infinity, state that explicitly instead of writing "does not exist." Answers with little work will be accepted here but the wrong answer with no work will receive no partial credit.

$4-6+2=0$

(a) $\lim_{x\to -2} \frac{x^2+3x+2}{x^2-x-6} = \lim_{x\to -2} \frac{(x+2)(x+1)}{(x+2)(x-3)}$ answer: $1/5$

$4+2-6=0$

$= \lim_{x\to -2} \frac{x+1}{x-3} = \frac{-1}{-5} = \frac{1}{5}$

(b) $\lim_{t\to -2^+} \frac{t-2}{2+t} = -\infty$ answer: $-\infty$

as $t \to -2^+$, (like -1.99) $t-2 \to -4$

and $2+t \to 0^+$

$\infty - \infty$

(c) $\lim_{x\to\infty} [\ln(2x) - \ln(5x+2)]$ answer: $\ln\left(\frac{2}{5}\right)$

$= \lim_{x\to\infty} \ln\left(\frac{2x}{5x+2}\right) = \ln\left[\lim_{x\to\infty}\left(\frac{2x}{5x+2}\right)\right] = \ln\left(\frac{2}{5}\right)$

(d) $\lim_{x\to 4} \frac{\frac{1}{x^2}-\frac{1}{16}}{x-4} = \lim_{x\to 4}\left(\frac{1}{x-4}\right)\left(\frac{16-x^2}{16x^2}\right)$ answer: $-1/32$

$= \lim_{x\to 4} \frac{(4-x)(4+x)}{-(4-x)(16x^2)} = \lim_{x\to 4} \frac{-(4+x)}{16x^2} = \frac{-8}{16\cdot 16} = \frac{-1}{32}$

8. (10 points) The temperature C, in degrees Celsius, was measured t seconds after a hot water faucet was turned on.

(a) It is found that $C(2) = 10$. Interpret this fact in the context of the problem. (That is, what does $C(2) = 10$ mean?) Include units in your answer.

Two seconds after the faucet was turned on the water temperature was $10\,°C$.

(b) It is found that $C'(2) = 3$. Interpret this fact in the context of the problem. (That is, what does $C'(2) = 3$ mean?) Include units in your answer.

After two seconds the rate of change of water temperature was $3\,°C$ per second.

(c) It is found that $\lim_{t\to\infty} C(t) = 40$. Interpret this fact in the context of the problem. (That is, what does $\lim_{t\to\infty} C(t) = 40$ mean?) Include units in your answer.

As the water faucet is left on, the temperature of the water approaches $40\,°C$. $\boxed{\text{OR}}$ In the long term, the temperature of the water gets close to $40\,°C$

Extra Credit (5 points) Use the Intermediate Value Theorem to prove that there exists some x-value in the interval $(0, 100)$ such that the function $g(x) = 15e^{-x} - \sqrt{x} + 3$ takes the value $y = 10$.

Observe that $g(x)$ is continuous for all real numbers.

Thus, IVT applies.

$g(0) = 15e^{0} - 0 + 3 = 18 > 10.$

$g(100) = 15e^{-100} - \sqrt{100} + 3 = \frac{15}{e^{100}} - 7 < 10$

Since g is continuous on $[0,100]$ and larger than 10 at one end $(x=0)$ and smaller than 10 at the other end $(x=100)$, the IVT says g must equal 10 somewhere in between $x=0$ and $x=100$.