Math F251 Final Exam Fall 2021

Name: _____

Section: ☐ **F01 (Faudree)**
☐ **F02 (Gossell)**
☐ **UX1 (Gossell)**

Rules:

You have 2 hours to complete the exam.

Partial credit will be awarded, but you must show your work.

No other aids are permitted.

Place a box around your FINAL ANSWER to each question where appropriate.

Turn off anything that might go beep during the exam.

Problem	Possible	Score
1	8	
2	8	
3	10	
4	8	
5	10	
6	10	
7	6	
8	6	
9	5	
10	5	
11	12	
12	12	
Extra Credit	5	
Total	100	

1. (8 points)

Find the derivative of each of the following functions. You do not need to simplify your answer.

a. $f(x) = (\cos x)(\ln(x^2 + 1))$

$f'(x) =$

b. $g(x) = e^{\sqrt{x}} + 5x^3 + \sin\left(\frac{\pi}{4}\right)$

$g'(x) =$

2. (8 points)

Evaluate the following indefinite integrals.

a. $\int \left(2x^4 - \frac{4}{x^2}\right) dx$

b. $\int \frac{\sec^2 x}{\tan x} dx$

3. (10 points)

Evaluate the following limits. If you use L'Hopital's Rule, please indicate the form of the limit ($0/0$ or ∞/∞).

a. $\lim_{x\to 6} \dfrac{\frac{1}{6} - \frac{1}{x}}{x - 36}$.

b. $\lim_{x\to 3^-} \dfrac{2x^2 - 3x}{x^2 - 7x + 12}$

4. (8 points)

The temperature in a cabin is given by

$$T(t) = 55 + \frac{20t}{t+1}$$

where T is measured in degrees Fahrenheit and $t \geq 0$ is measured in minutes after starting the wood stove.

a. At what **rate** is the temperature changing at time $t = 0$? Include units in your answer.

b. Compute $\lim_{t\to\infty} T(t)$ and explain what this number means in language the general public might understand.

5. (10 points)

A drone is launched off a 2-foot platform. Its upward velocity in feet per second at t seconds is measured by the function $v(t) = \frac{3}{1+t^2}$.

a. Find $h(t)$, the height of the drone at t seconds.

b. Find $h(1)$, the height after 1 second. Include units. Hint: $\tan(\pi/4) = 1$.

c. Find $a(t)$, the acceleration function at t seconds.

d. Find $a(1)$, the acceleration after 1 second. Include units.

e. Find $v(1)$, the upward velocity after 1 second. Include units.

f. Use your answers from parts (d) and (e) above to determine whether the drone speeding up or slowing down when $t = 1$.

6. (10 points)

A manufacturer discovers that the revenue gained by producing and selling x products is $R(x) = 420x$ and the cost is $C(x) = 2x^2 + 5000$.

a. Write the profit function, $P(x)$. (Hint: Remember that profit is revenue minus cost: $P(x) = R(x) - C(x)$.)

b. What is the domain of the profit function?

c. How many products should the manufacturer produce to maximize the profit? Be sure to justify that your answer is correct. That is, use Calculus to show that your answer indeed does represent a maximum or minimum.

7. (6 points)

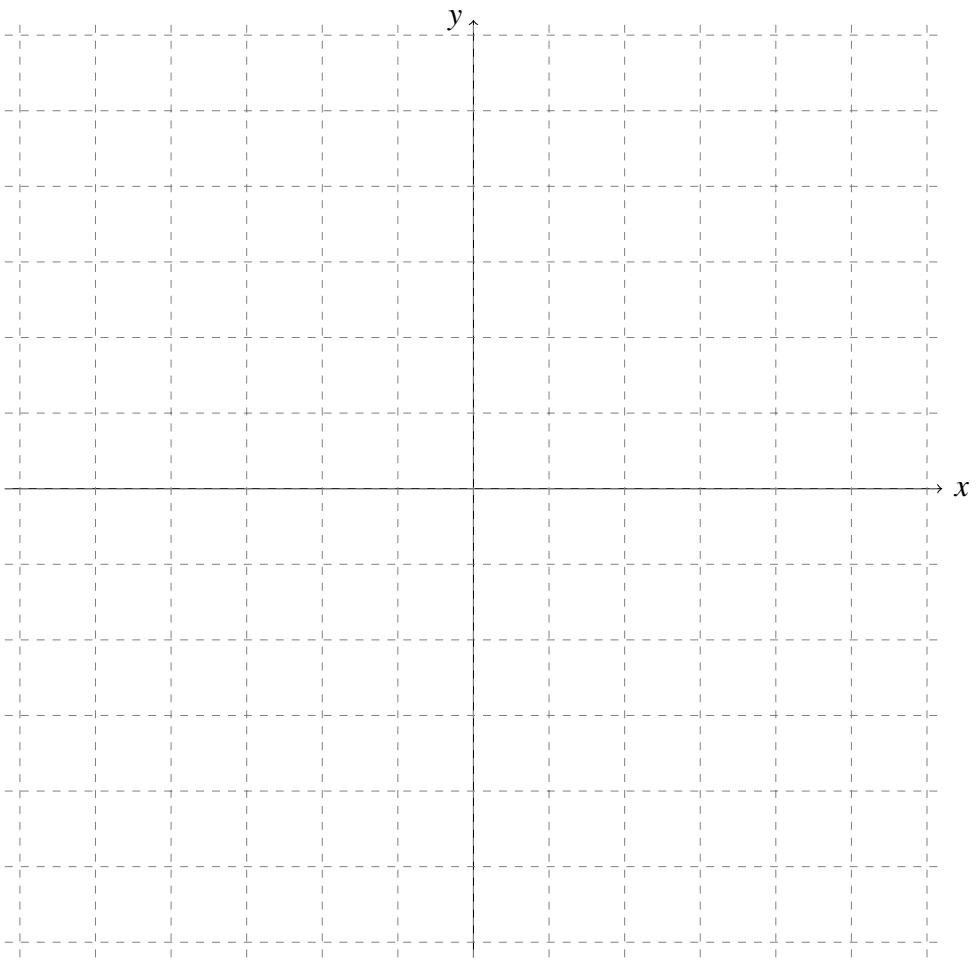
The volume of a spherical balloon is given by $V = \frac{4}{3}\pi r^3$. If the balloon is being inflated at a rate of 32π cubic inches per minute, how fast is the balloon's radius changing when the radius is 2 inches? Give units with your answer.

8. (6 points)

The function $G(x)$ is continuous on its domain $(-\infty, 0) \cup (0, \infty)$.

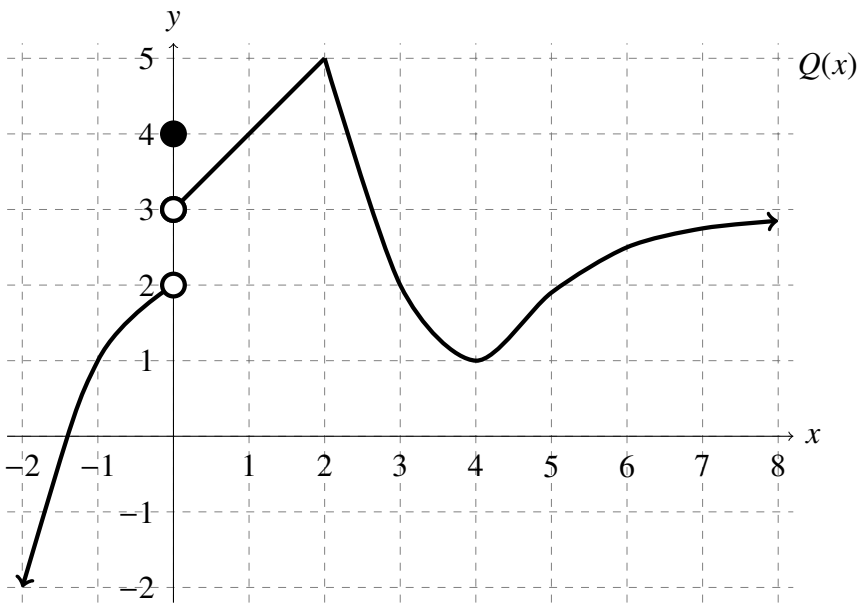
- $G'(x)$ is negative for all x in the domain $(-\infty, 0) \cup (0, \infty)$.
- $G''(x)$ is negative in the interval $(-2, 0)$
- $G''(x)$ is positive in the interval $(-\infty, -2) \cup (0, \infty)$.
- $\lim_{x\to\infty} G(x) = 1$.
- $\lim_{x\to 0^+} G(x) = \infty$ and $\lim_{x\to 0^-} G(x) = -\infty$

Sketch the graph of $G(x)$.



9. (5 points)

Consider the function $Q(x)$ graphed below.



a. Find $\lim_{x \to 0^-} Q(x)$

b. Find $\lim_{x \to 4} Q(x)$

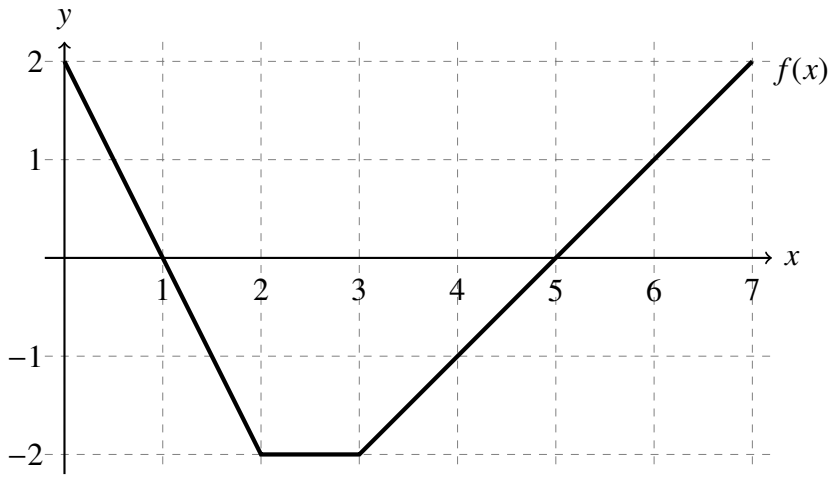
c. Find $\lim_{x \to \infty} Q(x)$

d. For what values of x, if any, does $Q(x)$ fail to be continuous?

e. At what values of x, if any, does $Q'(x)$ not exist?

10. (5 points)

Consider the function $f(x)$ graphed below.



a. What is the value of $f(1)$?

b. What is the value of $f'(1)$?

The following questions concern $H(x) = \int_0^x f(s)\, ds$.

c. What is the value of $H(7)$?

d. What is the value of $H'(4)$?

e. At what values of x does $H(x)$ have a local minimum?

11. (12 points)

The following questions concern $f(x) = \dfrac{x^2 - 1}{(x-3)^2}$. Note $f'(x) = -\dfrac{2(3x-1)}{(x-3)^3}$ and $f''(x) = \dfrac{12(x+1)}{(x-3)^4}$.

a. Find any critical points of $f(x)$.

b. Identify the locations of any local minimums or local maximums. Justify your conclusions. If no local minimum or local maximum exists, state this explicitly.

c. Does $f(x)$ have any inflection points? Justify your conclusions.

12. (12 points)

Water flows into a tank at a rate of $r(t) = 6 + 4t - t^2$ liters per minute from $t = 0$ to $t = 5$ minutes.

a. Compute $\int_0^2 r(t)\,dt$.

b. Interpret your answer from part (a) in the context of the problem. Make sure to include units.

c. At time $t = 0$, the tank contained 50 liters of water. How much water is in the tank at time $t = 2$?

13. (Extra Credit: 5 points)

Evaluate $\lim_{x \to 0^+} \tan x \ln x$. You must show your work.