Math F251 Midterm 2 Fall 2022

Name: ____________________

Rules:

You have 90 minutes to complete the exam.

Partial credit will be awarded, but you must show your work.

You may have a single handwritten 3×5 notecard.

Calculators are not allowed.

Place a box around your FINAL ANSWER to each question where appropriate.

Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	10	
2	10	
3	10	
4	10	
5	18	
6	10	
7	10	
8	10	
9	12	
Extra Credit	5	
Total	100	

1. (10 points) Evaluate the following limits. **You must show your work and justify your answer to earn full credit.** If you apply L'Hopital's Rule, you should indicate this, by writing $\stackrel{H}{=}$ or $\stackrel{L'H}{=}$ or some other clear indication.

   (a) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x^2+4}}{5x+4}$

   (b) $\displaystyle\lim_{x\to 0} \frac{x^2}{3-3\cos(x)}$

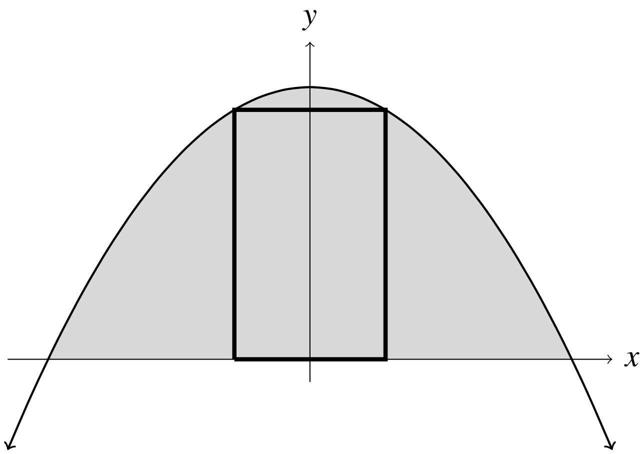
2. (10 points)

   (a) Find the linear approximation (also known as the linearization) of the function $f(x) = \sqrt{x}$ when $a = 1$.

   (b) Use the linear approximation from part (a) to estimate $\sqrt{1.05}$. Your answer must be in the form of a simplified decimal or an exact fraction.

3. (10 points) The formula for the volume, V, of a cone in terms of its radius r and height h is $V = \frac{1}{3}\pi r^2 h$. If the volume of the cone remains **constant** and the radius of the cone is increasing at a rate of 2 cm/s, determine the rate of change of the height of the cone at the instant the radius is 10 cm and the height is 20 cm. **Interpret your answer using a compete sentence.**

4. (10 points) Determine the maximum area of a rectangle with base on the x-axis inscribed between the parabola $y = 12 - x^2$ and the x-axis (See figure below.) Note: Your solution must use Calculus to **justify** that your answer is correct.



answer: Maximum area is ________________________

5. (18 points) Use the information below to answer questions about the function $f(x)$. You must show your work to earn full credit.

$$f(x) = \frac{x}{e^x}, \quad f'(x) = \frac{-(x-1)}{e^x}, \quad f''(x) = \frac{x-2}{e^x}.$$

(a) Determine the intervals on which $f(x)$ is increasing/decreasing.

(b) Find the x-values that correspond to any local maximums or local minimums of $f(x)$.

(c) Find the intervals on which $f(x)$ is concave up and concave down.

(d) Find the x-values of any inflection points of $f(x)$. If there aren't any, you must explicitly state this and justify your answer.

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Note the function and its derivatives are:

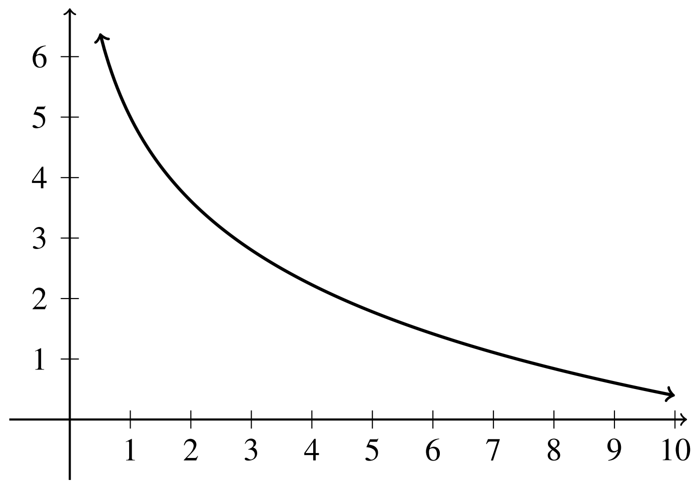
$$f(x) = \frac{x}{e^x}, \quad f'(x) = \frac{-(x-1)}{e^x}, \quad f''(x) = \frac{x-2}{e^x}.$$

(e) Give the equation of any horizontal asymptotes of $f(x)$ or state that none exist. Justify your answer using Calculus.

(f) Give the equation of any vertical asymptotes of $f(x)$ or state that none exist. Justify your answer using using Calculus.

6. (10 points) The function $f(x) = 5 - 2\ln(x)$ is graphed below. We want to estimate the area under the curve $f(x)$ on the interval $[1, 9]$ using L_4. (That is, we want to use 4 approximating rectangles and left-hand end points.)

   (a) Sketch the four approximating rectangles on the graph.



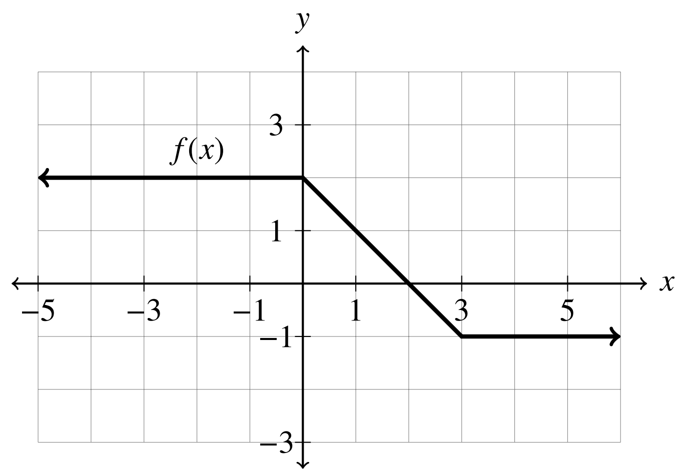
   (b) Do a calculation to estimate the area under the curve using L_4 (that is, use 4 approximating rectangles and left-hand end points) and simplify your answer. Note: You are obviously not expected to compute things like ln(4). It is acceptable to have numbers like this in your final answer.

7. (10 points) Evaluate the indefinite integrals below.

   (a) $\displaystyle\int (4\sin(x) + x^5 + x^{-1} + 10)\,dx$

   (b) $\displaystyle\int \frac{1 + x^3}{x^2}\,dx$

8. (10 points) Evaluate the definite integrals below using the graph of $f(x)$ and properties of definite integrals. Show your work.



   (a) $\displaystyle\int_{-2}^{4} f(x)\,dx$

   (b) $\displaystyle\int_{-2}^{4} (5f(x) + 3)\,dx$

9. (12 points) Sketch the graph of a function $f(x)$ that satisfies all of the given conditions. Clearly label any important points on the x-axis, draw any asymptotes clearly as dashed lines, and label any asymptotes with their equations.

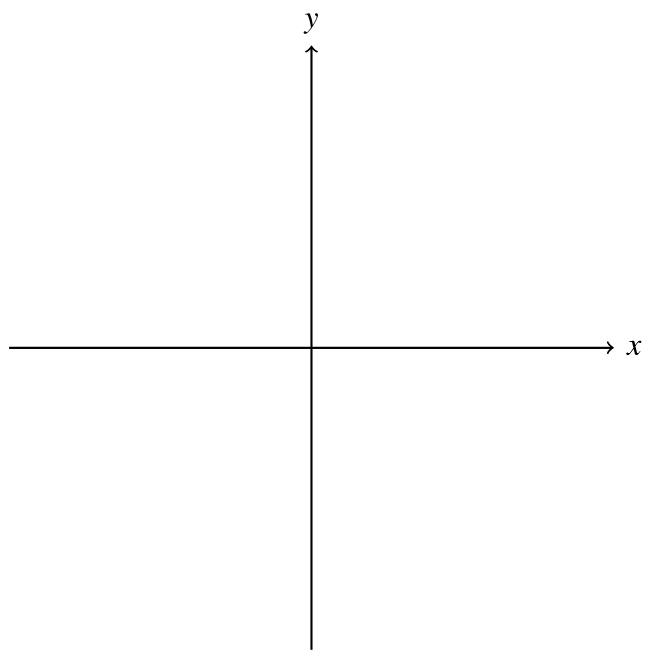
   (a) The domain of $f(x)$ is $(-\infty, \infty)$.

   (b) $f(1) = 0$ and $f'(1)$ is undefined

   (c) $\lim_{x \to -\infty} f(x) = 5$

   (d) $f'(x) < 0$ on the interval $(-\infty, 1)$; $f'(x) > 0$ on the interval $(1, \infty)$

   (e) $f''(x) < 0$ on the interval $(-\infty, 1)$; $f''(x) > 0$ on the interval $(1, \infty)$



Extra Credit: Suppose $C(t)$ models the position of a car and $B(t)$ models the position of a bike over the same time interval, $[0, 2]$, where C and B are measured in miles and t in hours.
(3 pts) Translate the following sentence into the language of Calculus: "The car goes faster than the bike but the bike accelerates faster than the car." (i.e. rewrite the sentence using derivatives in some form.)
(2pts) Construct a pair of functions $C(t)$ and $B(t)$ satisfying the properties described in the sentence on the interval $[0, 2]$. (Note, your functions do not have to be realistic...)