Fall 2025 **Math F251X**

Calculus 1: Midterm 1

Name: Key

Section:
- ☐ 9:15am (Kevin Meek)
- ☐ 11:45am (James Gossell)
- ☐ 11:45am (Margaret Short)
- ☐ async (James Gossell)

Rules:

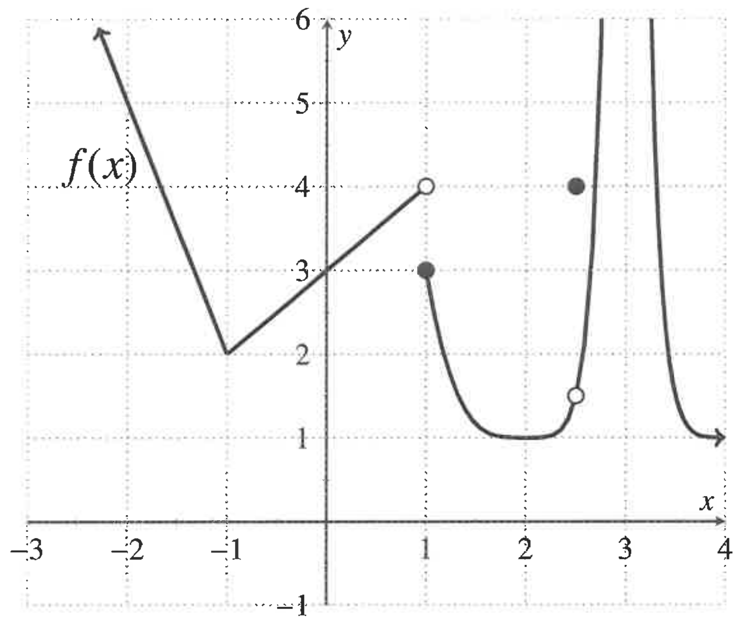
- Partial credit will be awarded, but you must show your work.
- You may have a single handwritten 3″ × 5″ notecard, both sides.
- Calculators are **not** allowed.
- Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.
- Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	16	
2	9	
3	9	
4	10	
5	8	
6	6	
7	12	
8	12	
9	18	
Extra Credit	(5)	
Total	100	

1. (16 points)

Consider the graph of the function $f(x)$ below.



Use the graph of $f(x)$ to answer each question below. If the limit is infinite, indicate that with ∞ or $-\infty$. If the value does not exist or is undefined, write DNE.

(a) $\lim_{x\to 1^-} f(x) =$ 4

(b) $\lim_{x\to 1^+} f(x) =$ 3

(c) $\lim_{x\to 1} f(x) =$ DNE

(d) $\lim_{h\to 0} \dfrac{f(0+h)-f(0)}{h} =$ 1

(e) $\lim_{x\to 2} f'(x) =$ 0

(f) $\lim_{x\to -2} f'(x) =$ −3

(g) $f(2.5) =$ 4

(h) $\lim_{x\to 2.5} f(x) =$ 1.5

(i) $\lim_{x\to 3} f(x) =$ $+\infty$

(j) Indicate **all** x-values for which the function $f(x)$ is **not continuous**:

1, 2.5, 3

(k) Indicate **all** x-values for which the function $f(x)$ is **not differentiable**:

−1, 1, 2.5, 3

2. (9 points)

Compute the following limits. Show your work. Use limit notation where necessary; you will be graded both on your computation and on your correct use of notation.

If the limit does not exist, write **DNE** and a few words about why it does not exist. If the limit increases without bound, write ∞ or $-\infty$.

a. $\lim\limits_{x\to4^-} \dfrac{x^2-8x+15}{x^2-9x+20} = \lim\limits_{x\to4^-} \dfrac{(x-3)\cancel{(x-5)}}{(x-4)\cancel{(x-5)}} = \lim\limits_{x\to4^-} \dfrac{x-3}{x-4} \to \dfrac{1}{0^-} \to \boxed{-\infty}$

b. $\lim\limits_{x\to2} \dfrac{\frac{2}{x}-\frac{x}{2}}{x-2} = \lim\limits_{x\to2} \dfrac{\frac{4}{2x}-\frac{x^2}{2x}}{x-2} = \lim\limits_{x\to2} \dfrac{\frac{4-x^2}{2x}}{x-2} = \lim\limits_{x\to2} \dfrac{\frac{(2-x)(2+x)}{2x}}{x-2}$

$\frac{0}{0}$

$= \lim\limits_{x\to2} \dfrac{-\cancel{(x-2)}(x+2)}{2x\cancel{(x-2)}} = \lim\limits_{x\to2} \dfrac{-x-2}{2x} = \dfrac{-4}{4} = \boxed{-1}$

c. $\lim\limits_{\theta\to\pi/4} \dfrac{\sin^2\theta}{\cos\theta} = \dfrac{\sin^2\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} = \dfrac{\left(\frac{\sqrt{2}}{2}\right)^2}{\left(\frac{\sqrt{2}}{2}\right)} = \boxed{\dfrac{\sqrt{2}}{2}}$

↑

continuous at $\frac{\pi}{4}$

3. (9 points)

Let

$$f(x) = \begin{cases} \dfrac{5x^2 + 2x}{x} & x < 0 \\ 5 & x = 0 \\ 5\sin(x) + 2\cos(x) & x > 0 \end{cases}$$

a. Evaluate $\lim_{x \to 0^-} f(x)$. Show supporting work.

$$\lim_{x \to 0^-} \frac{5x^2 + 2x}{x} = \lim_{x \to 0^-} (5x + 2) = 0 + 2 = \boxed{2}$$

b. Evaluate $\lim_{x \to 0^+} f(x)$. Show supporting work.

$$\lim_{x \to 0^+} (5\sin x + 2\cos x) = 5(0) + 2(1) = \boxed{2}$$

c. Evaluate $f(0)$.

$\boxed{5}$

d. Based on your answers to parts (a), (b) and (c), **check the true statement(s) below**:

- ☐ f is continuous at $x = 0$.
- ☑ f has a removable discontinuity at $x = 0$.
- ☐ f has a jump discontinuity at $x = 0$.
- ☐ f has an infinite discontinuity at $x = 0$.
- ☐ None of the above.

4. (10 points)

Use the limit definition of the derivative (given below) to find the derivative of

$$f(x) = \sqrt{x+3}$$

Show all your work clearly using correct notation. **No credit will be awarded for a solution without adequate justification or that does not use the definition below.**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

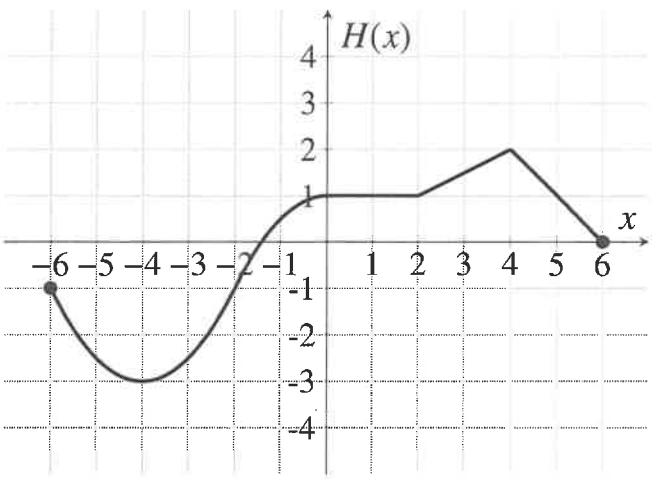
$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h} \cdot \frac{(\sqrt{x+h+3} + \sqrt{x+3})}{(\sqrt{x+h+3} + \sqrt{x+3})}$$

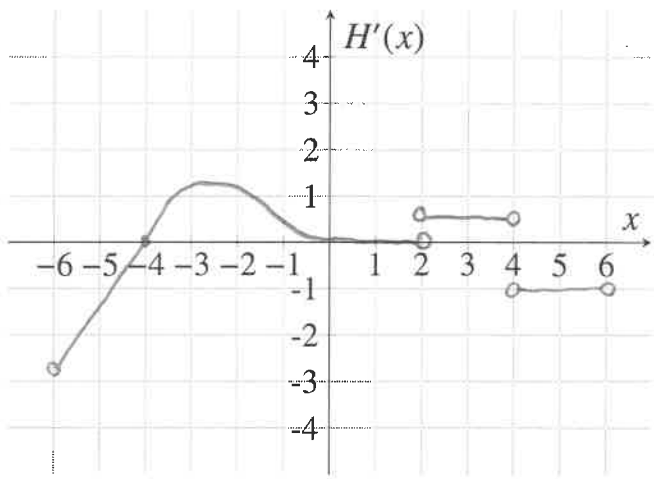
$$= \lim_{h \to 0} \frac{x+h+3-x-3}{h(\sqrt{x+h+3} + \sqrt{x+3})} = \lim_{h \to 0} \frac{h}{h(\sqrt{x+h+3} + \sqrt{x+3})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h+3} + \sqrt{x+3}} = \frac{1}{\sqrt{x+3} + \sqrt{x+3}} = \frac{1}{2\sqrt{x+3}}$$

5. (8 points)

The function $y = H(x)$ is graphed below. Sketch the graph of $H'(x)$ on the blank set of axes provided.





6. (6 points)

$S(t)$ is a function that describes the number of survivors in UAF's annual Humans vs. Zombies game, t days after the start of the game.

a. Interpret the meaning of $\boxed{S(2) = 235}$ in the context of the problem. Write a sentence or two including appropriate units.

After 2 days, there are 235 survivors.

b. Interpret the meaning of $\boxed{S'(2) = -53}$ in the context of the problem. Write a sentence or two including appropriate units.

2 days after the start, the number of survivors is decreasing at a rate of 53 survivors per day.

c. Using parts (a) and (b), **estimate** $S(3)$ including units. Write a sentence explaining how you arrived at your estimation.

$S(3) \approx S(2) + S'(2) = 235 - 53 = 182$ survivors.

$S(2) = 235$ survivors and since the number of survivors is decreasing by 53 survivors per day, it's likely that $S(3)$ will be about 182.

7. (12 points)

Compute the **derivative** of each of the following functions using any method you like. **You do not need to simplify your answers.**

a. $g(x) = x^{3.4} - \dfrac{2}{3x^2} + \sqrt{x} - \sin\left(\dfrac{\pi}{2}\right)$ $= x^{3.4} - \frac{2}{3}x^{-2} + x^{\frac{1}{2}} - \sin\left(\frac{\pi}{2}\right)$

$$\boxed{g'(x) = 3.4x^{2.4} + \frac{4}{3}x^{-3} + \frac{1}{2}x^{-\frac{1}{2}}}$$

b. $f(x) = \cos(x)\left(4x^5 - 3x^2\right)$

$$\boxed{f'(x) = -\sin x\left(4x^5 - 3x^2\right) + \cos x\left(20x^4 - 6x\right)}$$

c. $h(\theta) = \dfrac{\sin(\theta)}{\theta}$

$$\boxed{h'(\theta) = \frac{\cos(\theta)\cdot\theta - \sin(\theta)}{\theta^2}}$$

d. $s(x) = x^2(\sin x)(\cos x)$

$$\boxed{s'(x) = 2x\left(\sin x\cdot\cos x\right) + x^2\left(\cos x\cdot\cos x + \sin x\,(-\sin x)\right)}$$

9. (18 points)

An NFL quarterback throws a football straight up from the surface of the moon. While the football is in the air, its height in meters after t seconds is $h(t) = t(28 - 0.8t)$ $h(t) = -0.8t^2 + 28t$

a. Find functions describing the **velocity** and **acceleration** of the football.

$v(t) = h'(t) = -1.6t + 28$

$a(t) = h''(t) = -1.6$

b. What was the **initial velocity** of the football? Include correct units in your answer.

$v(0) = \boxed{28 \text{ meters per second}}$

c. After 10 seconds, is the football speeding up or slowing down? Show your work and justify your answer.

$v(10) = -16 + 28 = 12$ The football is slowing down because $v(10) > 0$ but $a(10) < 0$

$a(10) = -1.6$

d. When does the football reach its highest point? Include correct units in your answer.

Set $v(t) = 0$: $-1.6t + 28 = 0 \Rightarrow t = \frac{-28}{-1.6} = \frac{280}{16} = \frac{35}{2} = \boxed{17.5 \text{ seconds}}$

e. When does the fooball hit the ground? Include correct units in your answer.

Set $h(t) = 0$: $t(28 - 0.8t) = 0 \Rightarrow t = 0$ OR $28 - 0.8t = 0 \Rightarrow t = \frac{-28}{-0.8} = \frac{280}{8} = 35$

start time

end of flight

$\boxed{35 \text{ seconds}}$

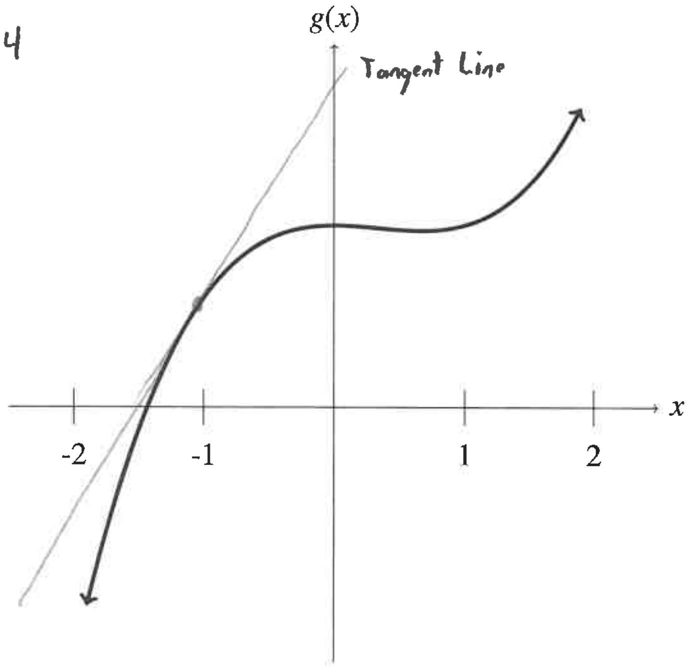
f. What is the **acceleration due to gravity** on the surface of the moon? Include correct units in your answer.

$\boxed{-1.6 \text{ meters per second}^2}$

8. (12 points)

The function $g(x) = x^2(x-1)+4$ is graphed below.

$g(x) = x^3 - x^2 + 4$



a. Sketch and label the graph of the **tangent line** to the graph of $g(x)$ when $x = -1$.

b. Write an equation for the tangent line to $g(x)$ when $x = -1$.

$g'(x) = 3x^2 - 2x$

$g'(-1) = 3(-1)^2 - 2(-1) = 5$

$g(-1) = (-1)^2(-1-1) + 4 = 2$

Tangent Line: $y - 2 = 5(x+1)$

$y = 5x + 5 + 2$

$\boxed{y = 5x + 7}$

c. Find all the x-values at which the graph of $g(x)$ has a horizontal tangent line or explain why none exist. Justify your answer and show your work.

Set $g'(x) = 0$: $3x^2 - 2x = 0 \Rightarrow x(3x-2) = 0 \Rightarrow x = 0$ OR $3x - 2 = 0$

$\Rightarrow x = \frac{2}{3}$

10. (Extra Credit: 5 points)

Is it always true that $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{f'(x)}{g'(x)}$?

If so, explain why. If not, provide an example where $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] \neq \dfrac{f'(x)}{g'(x)}$.

No. Consider $\dfrac{d}{dx}\left[\dfrac{x^3}{x^2}\right]$:

$\dfrac{d}{dx}\left[\dfrac{x^3}{x^2}\right] = \dfrac{d}{dx}\left[x\right] = 1$ which is not $\dfrac{\frac{d}{dx}\left[x^3\right]}{\frac{d}{dx}\left[x^2\right]} = \dfrac{3x^2}{2x} = \dfrac{3}{2}x$