

Fall 2025

Math F251X

Calculus 1: Midterm 1

Name: _____

Section: ☐ 9:15am (Kevin Meek)

☐ 11:45am (James Gossell)

☐ 11:45am (Margaret Short)

☐ async (James Gossell)

Rules:

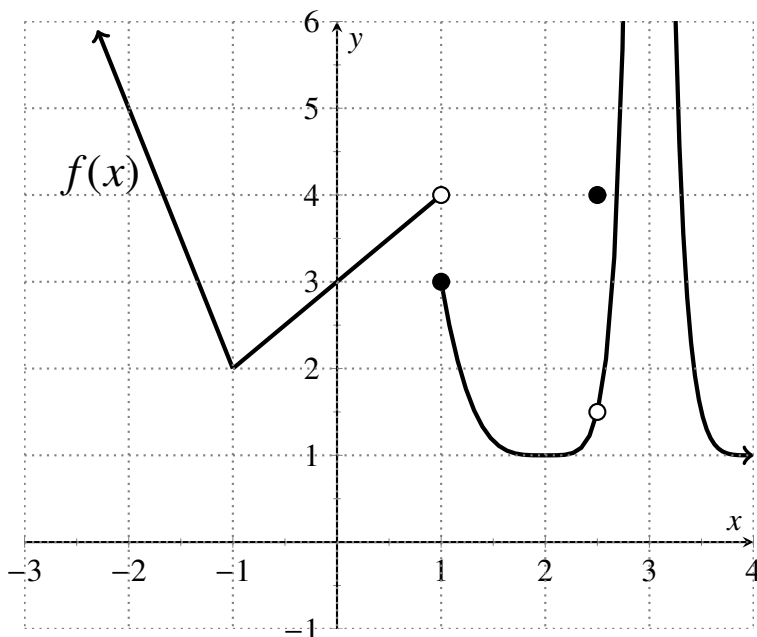
- Partial credit will be awarded, but you must show your work.
- You may have a single handwritten $3'' \times 5''$ notecard, both sides.
- Calculators are **not** allowed.
- Place a box around your FINAL ANSWER to each question where appropriate.
- Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	16	
2	9	
3	9	
4	10	
5	8	
6	6	
7	12	
8	12	
9	18	
Extra Credit	(5)	
Total	100	

1. (16 points)

Consider the graph of the function $f(x)$ below.



Use the graph of $f(x)$ to answer each question below. If the limit is infinite, indicate that with ∞ or $-\infty$. If the value does not exist or is undefined, write DNE.

(a) $\lim_{x \rightarrow 1^-} f(x) =$

(b) $\lim_{x \rightarrow 1^+} f(x) =$

(c) $\lim_{x \rightarrow 1} f(x) =$

(d) $\lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} =$

(e) $\lim_{x \rightarrow 2} f'(x) =$

(f) $\lim_{x \rightarrow -2} f'(x) =$

(g) $f(2.5) =$

(h) $\lim_{x \rightarrow 2.5} f(x) =$

(i) $\lim_{x \rightarrow 3} f(x) =$

(j) Indicate **all** x -values for which the function $f(x)$ is **not continuous**:

(k) Indicate **all** x -values for which the function $f(x)$ is **not differentiable**:

2. (9 points)

Compute the following limits. Show your work. Use limit notation where necessary; you will be graded both on your computation and on your correct use of notation.

If the limit does not exist, write **DNE** and a few words about why it does not exist. If the limit increases without bound, write ∞ or $-\infty$.

a. $\lim_{x \rightarrow 4^-} \frac{x^2 - 8x + 15}{x^2 - 9x + 20}$

b. $\lim_{x \rightarrow 2} \frac{\frac{2}{x} - \frac{x}{2}}{x - 2}$

c. $\lim_{\theta \rightarrow \pi/4} \frac{\sin^2 \theta}{\cos \theta}$

3. (9 points)

Let

$$f(x) = \begin{cases} \frac{5x^2 + 2x}{x} & x < 0 \\ 5 & x = 0 \\ 5 \sin(x) + 2 \cos(x) & x > 0 \end{cases}$$

a. Evaluate $\lim_{x \rightarrow 0^-} f(x)$. Show supporting work.

b. Evaluate $\lim_{x \rightarrow 0^+} f(x)$. Show supporting work.

c. Evaluate $f(0)$.

d. Based on your answers to parts (a), (b) and (c), **check the true statement(s) below:**

- ☐ f is continuous at $x = 0$.
- ☐ f has a removable discontinuity at $x = 0$.
- ☐ f has a jump discontinuity at $x = 0$.
- ☐ f has an infinite discontinuity at $x = 0$.
- ☐ None of the above.

4. (10 points)

Use the limit definition of the derivative (given below) to find the derivative of

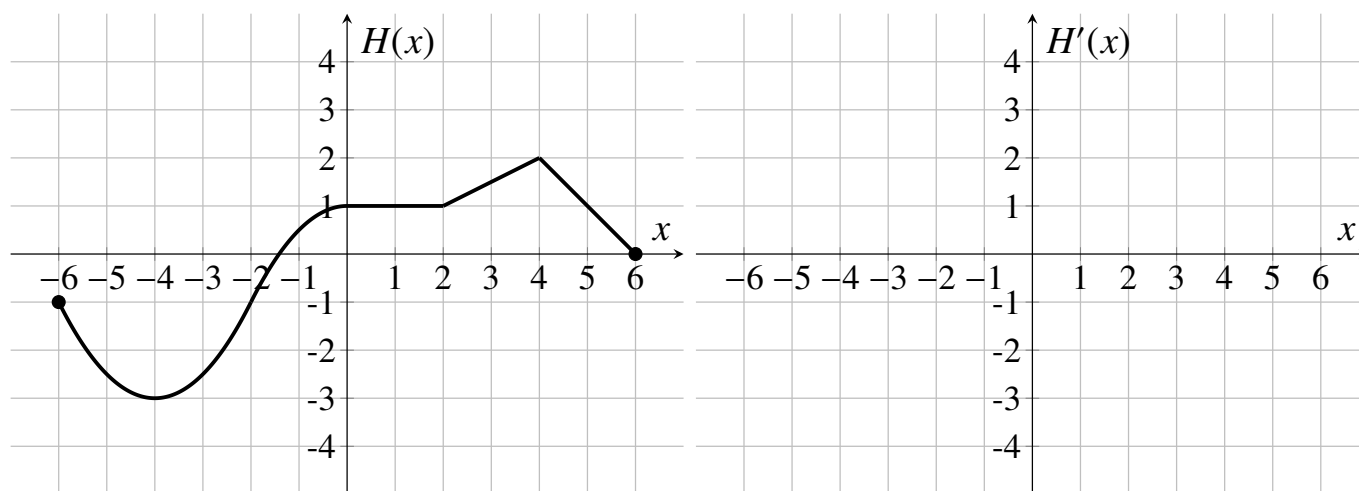
$$f(x) = \sqrt{x+3}$$

Show all your work clearly using correct notation. **No credit will be awarded for a solution without adequate justification or that does not use the definition below.**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

5. (8 points)

The function $y = H(x)$ is graphed below. Sketch the graph of $H'(x)$ on the blank set of axes provided.



6. (6 points)

$S(t)$ is a function that describes the number of survivors in UAF's annual Humans vs. Zombies game, t days after the start of the game.

- Interpret the meaning of $S(2) = 235$ in the context of the problem. Write a sentence or two including appropriate units.
- Interpret the meaning of $S'(2) = -53$ in the context of the problem. Write a sentence or two including appropriate units.
- Using parts (a) and (b), **estimate** $S(3)$ including units. Write a sentence explaining how you arrived at your estimation.

7. (12 points)

Compute the **derivative** of each of the following functions using any method you like. **You do not need to simplify your answers.**

a. $g(x) = x^{3.4} - \frac{2}{3x^2} + \sqrt{x} - \sin\left(\frac{\pi}{2}\right)$

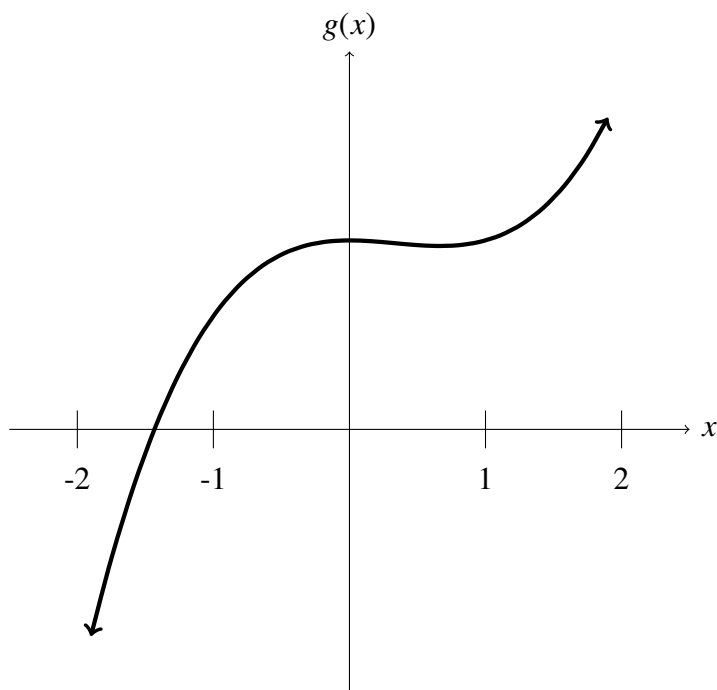
b. $f(x) = \cos(x)(4x^5 - 3x^2)$

c. $h(\theta) = \frac{\sin(\theta)}{\theta}$

d. $s(x) = x^2(\sin x)(\cos x)$

8. (12 points)

The function $g(x) = x^2(x - 1) + 4$ is graphed below.



- Sketch and label the graph of the **tangent line** to the graph of $g(x)$ when $x = -1$.
- Write an equation for the tangent line to $g(x)$ when $x = -1$.
- Find all the x -values at which the graph of $g(x)$ has a horizontal tangent line or explain why none exist. Justify your answer and show your work.

9. (18 points)

An NFL quarterback throws a football straight up from the surface of the moon. While the football is in the air, its height in meters after t seconds is $h(t) = t(28 - 0.8t)$

- a. Find functions describing the **velocity** and **acceleration** of the football.

- b. What was the **initial velocity** of the football? Include correct units in your answer.

- c. After 10 seconds, is the football speeding up or slowing down? Show your work and justify your answer.

- d. When does the football reach its highest point? Include correct units in your answer.

- e. When does the football hit the ground? Include correct units in your answer.

- f. What is the **acceleration due to gravity** on the surface of the moon? Include correct units in your answer.

10. (Extra Credit: 5 points)

Is it always true that $\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'(x)}{g'(x)}$?

If so, explain why. If not, provide an example where $\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] \neq \frac{f'(x)}{g'(x)}$.