Your Name

Your Signature

Instructor Name

Problem	Total Points	Score
1	16	
2	12	
3	6	
4	6	
5	8	
6	10	
7	12	
8	6	
9	10	
10	8	
11	6	
Total	100	

- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work.** Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- **PLACE A BOX AROUND** $\boxed{\textbf{YOUR FINAL ANSWER}}$ **to each question** where appropriate.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

1. (16 points) Find the derivative for each of the following functions. (For parts (a)-(c), you do not need to simplify your answers.)

   (a) $h(x) = \dfrac{\arctan(x)}{1+x}$

   (b) $f(x) = \sec\left(\sqrt{1-x^2}\right)$

   (c) $y = \dfrac{3}{x} + 3\ln(x) - \tan(3\pi)$

   (d) $2x^2 - 5xy + 4y^2 = 2$ (Solve for dy/dx.)

2. (12 points) Evaluate the following integrals.

(a) $g(x) = \int \left(\frac{2}{x} + 2x^{1/3} - e^2\right) \mathrm{d}x$

(b) $h(x) = \int 4\cos^3(x)\sin(x)\mathrm{d}x$

(c) $f(x) = \int \left(x\sqrt{2x-1}\right)\ \mathrm{d}x$

3. (6 points) Let $f(x) = \frac{1}{x}$.

   (a) Find the average rate of change of f from $x = 1$ to $x = 3$. Simplify your answer if possible.

   (b) Find $f'(x)$ using the definition of the derivative.

4. (6 points) Let $f(x) = x^{2/3}$.

   (a) Find the linearization $L(x)$ of $f(x)$ at $x = 8$.

   (b) Use your answer in part a to estimate $(8.1)^{2/3}$. Write your answer as a common fraction.

5. (8 points) The height, h, of water in a ditch is given by

$$h(t) = \frac{2 + \sin(\pi t)}{1 + t},$$

where h is measured in feet and t is measured in days.

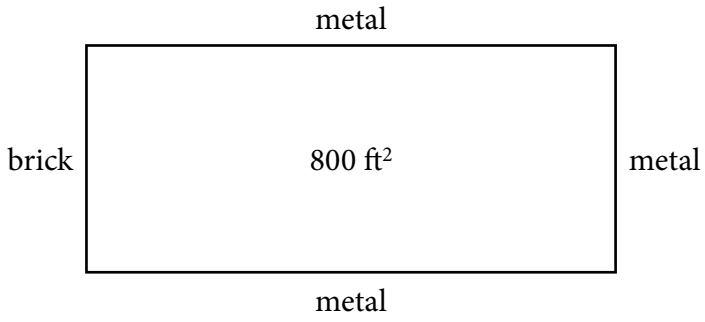
(a) Find and **interpret** $h(3)$ in the context of the problem. (Your expression for $h(3)$ should be simplified.)

(b) Find $h'(t)$. (You do not need to simplify your answer.)

(c) Interpret $h'(3) \approx -0.91$ in the context of the problem.

(d) Find and **interpret** $\lim_{t \to \infty} h(t)$. (Hint: $-1 \leq \sin(x) \leq 1$.)

6. (10 points) A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing \$30 per foot and on the other three sides with a metal fence costing \$10 per foot. The area of the garden is to be 800ft^2. What are the dimensions of the garden that minimize the cost of the fencing? (For full credit, you must justify your answer.)



7. (12 points) Let $g(x) = \dfrac{e^x}{1+x}$. Note first and second derivatives are

$$g'(x) = \frac{xe^x}{(1+x)^2} \quad \text{and} \quad g''(x) = \frac{e^x(x^2+1)}{(1+x)^3}.$$

(a) Evaluate the following limits.

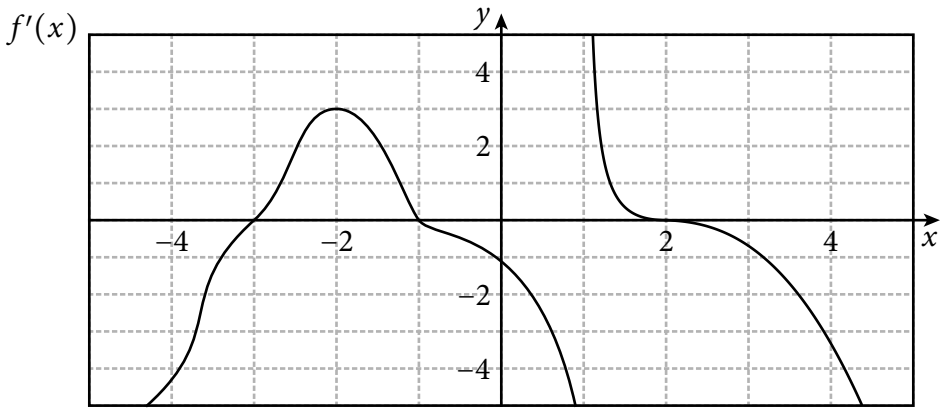
i. $\lim_{x\to\infty} g(x)$

ii. $\lim_{x\to-\infty} g(x)$

iii. $\lim_{x\to-1^-} g(x)$

(b) Sketch the graph of $g(x)$. Label any asymptotes, x- and y-intercepts, local minimums and local maximums, and inflection points, if appropriate.

8. (6 points) The graph of **the derivative** of $f(x)$, $f'(x)$, is shown below. Questions (a) through (d) concern the function $f(x)$.



(a) For what intervals(s) is $f(x)$ increasing?

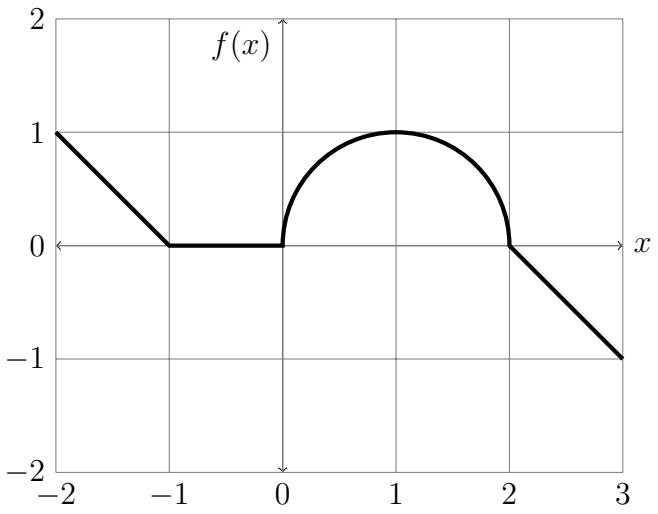
(b) For what intervals(s) is $f(x)$ concave up?

(c) What value(s) of x give $f(x)$ a relative maximum?

(d) What value(s) of x give $f(x)$ inflection points?

9. (10 points) The function $f(x)$ has been graphed below. The curve for $0 < x < 2$ is an upper half circle. Define a new function $g(x)$, as

$$g(x) = \int_0^x f(s)\, ds.$$



Use the graph above to answer the questions below.
Note: Pay attention to whether question concerns the function f, f', g or g'.

(a) What is the value of $f(0)$?

(b) What is the value of $g(3)$?

(c) What is the value of $g(-2)$?

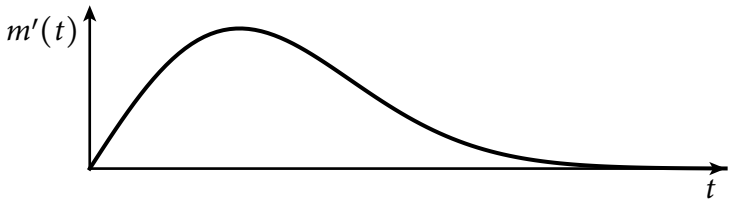
(d) What is the value of $f'(2)$?

(e) What is the value of $g'(1)$?

10. (8 points) Snow is accumulating on my deck. The total amount of snow on my deck is $m(t)$ kilograms, where $t > 0$ is measured in hours. The instantaneous rate of accumulation is

$$m'(t) = 4te^{-t^2}$$

kilograms per hour.

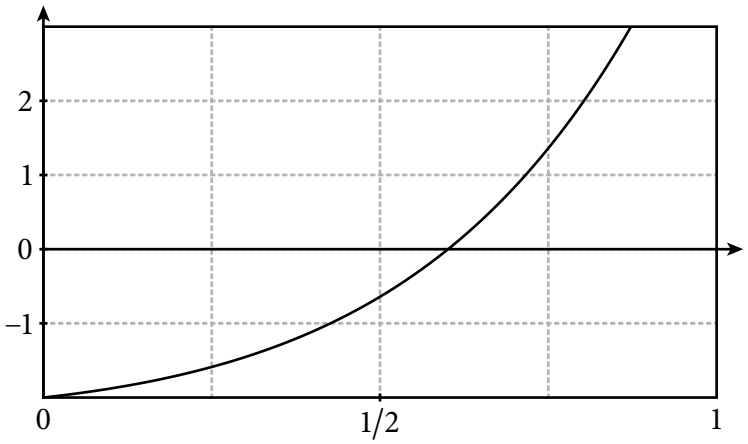


(a) At what time is the **rate** of snow accumulation at its peak?

(b) In the diagram above, label the time, t, obtained in part (a).

(c) Assume that at time $t = 0$ there are 10kg of snow on the deck. How much snow is on the deck at time $t = 2$ hours?

11. (6 points) Consider the function $f(x) = xe^{2x} - 2$ graphed below.



(a) Approximate the solution of $f(x) = 0$ using **JUST ONE** iteration of Newton's method starting from an initial guess of $x_0 = 1/2$ to compute a new estimate: x_1. It is OK to leave your answer unsimplified, but your answer should be an expression you could compute if you had a calculator.

(b) In the figure above, indicate the point x_1 you computed in part (b) and demonstrate in the diagram how x_1 was obtained from x_0.