Your Name

Your Signature

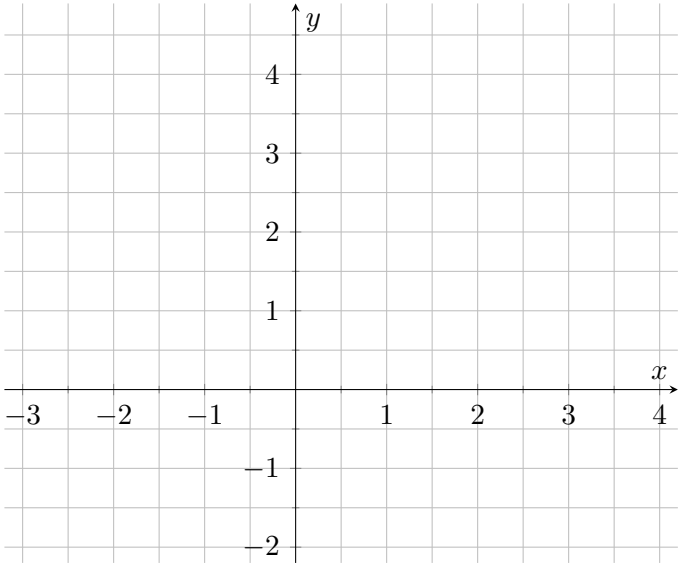
Instructor Name

Problem	Total Points	Score
1	20	
2	21	
3	20	
4	12	
5	15	
6	12	
Extra Credit	(5)	
Total	100	

- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work.** Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- **PLACE A BOX AROUND $\boxed{\text{YOUR FINAL ANSWER}}$ to each question** where appropriate.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

1. Given the function $f(x) = \begin{cases} 3 - x^2 & x \leq 1 \\ \sin(\pi x) & x > 1 \end{cases}$

(a) (8 points) Sketch the graph of $f(x)$



(b) (2 points each) Evaluate each of the limits below.

i. Find $\lim_{x \to 1^+} f(x)$

ii. Find $\lim_{x \to 1^-} f(x)$

iii. Find $\lim_{x \to \infty} f(x)$

iv. Find $\lim_{x \to -\infty} f(x)$

(c) (4 points) Determine whether the function is continuous at $x = 1$ and justify your answer using the definition of continuity.

2. (21 points) Evaluate the following limits and justify your answers.

(a) $\lim_{x \to -1} \dfrac{x^2 + 4x + 3}{x^2 + x}$

(b) $\lim_{x \to 5^+} \dfrac{x^2 + 2}{5 - x}$

(c) $\lim_{x \to \infty} \dfrac{3 - 5e^x}{2e^x + 1}$

(d) $\lim_{x \to -17} |x + 15|$

3. (20 points) A population of moose is declining. The population at time t is

$$P(t) = \frac{1000}{1+t}$$

where P is the number of moose and t is measured in years.

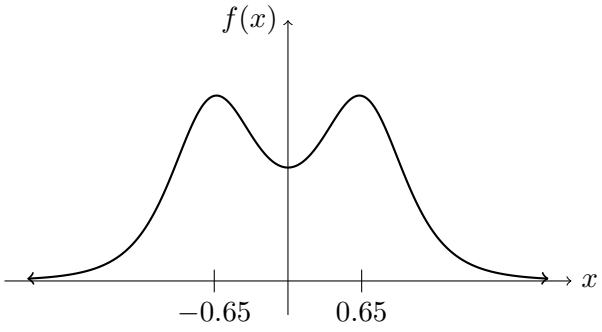
(a) (5 points) Compute the average rate of change of the population from time $t = 1$ to $t = 4$ years.

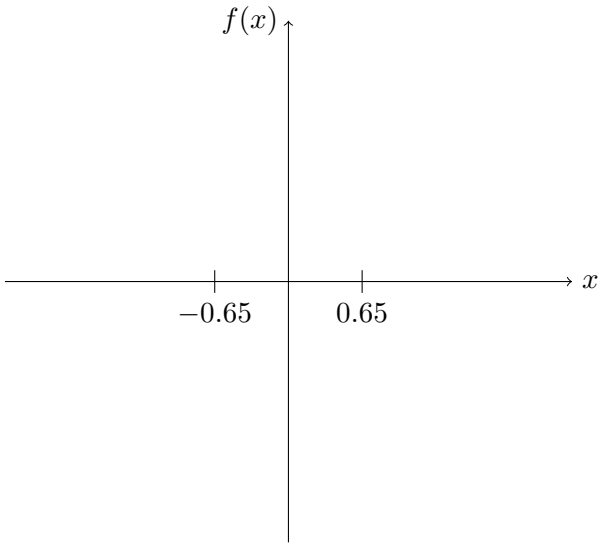
(b) (5 points) Compute the average rate of change of the population from time $t = 1$ to $t = b$ years.

(*part (c) continued on next page* $\longrightarrow$)

(c) (10 points) Using a limit, find the instantaneous rate of change of the moose population at time $t = 1$ years. No credit will be given for using a derivative rule you may have learned in the past. You must compute the rate of change from the limit definition.

4. (12 points) The axis on the left is the graph of a function $f(x)$. In the axis on the right, sketch the graph of $f'(x)$.





5. (15 points) A cup of coffee is cooling on a desk. The temperature of the coffee is

$$T(t) = 20 + 80 \cdot 10^{-t/30}.$$

where t is measured in minutes from some initial time and T is measured in degrees Celsius.

(a) At what time t will the coffee's temperature be equal to 60°C?

(b) In the context of the problem, interpret $T'(t)$. (Include units.)

(c) What does the statement $T'(10) = -2.85$ mean?

6. (12 points) Assume $f(x) = \frac{6x}{x+2}$ and $f'(x) = \frac{12}{(x+2)^2}$. Find the equation of the line tangent to the graph $f(x)$ when $x = 2$.

Extra Credit (5 points) Use the Intermediate Value Theorem to show that the two curves $f(x) = 4x + \sin x$ and $g(x) = x - e^{-x}$ must intersect.