Your Name

Your Signature

Instructor Name

Problem	Total Points	Score
1	15	
2	15	
3	10	
4	20	
5	15	
6	15	
7	10	
Extra Credit	(5)	
Total	100	

- This test is closed notes and closed book.

- You may **not** use a calculator.

- In order to receive full credit, you must **show your work.** Be wary of doing computations in your head. Instead, write out your computations on the exam paper.

- **PLACE A BOX AROUND** YOUR FINAL ANSWER **to each question** where appropriate.

- If you need more room, use the backs of the pages and indicate to the reader that you have done so.

- Raise your hand if you have a question.

Geometric Formulas

sphere
$V = \frac{4}{3}\pi r^3$
$A = 4\pi r^2$

cylinder
$V = \pi r^2 h$

cone
$V = \frac{1}{3}\pi r^2 h$
$A = \pi r \sqrt{r^2 + h^2}$

1. (15 points) Consider the function $f(x) = \dfrac{(x-1)^3}{x^2}$. We have computed for you

$$f'(x) = \frac{(x-1)^2(x+2)}{x^3} \quad \text{and} \quad f''(x) = \frac{6x-6}{x^4}.$$

For full credit, show your work.

(a) Find the intervals where $f(x)$ is increasing and decreasing.

(b) Find the intervals where $f(x)$ is concave up and concave down.

(c) Classify all critical points of $f(x)$.

2. (15 points) Evaluate the following limits. [Note: You should be careful to apply L'Hospital's rule *only* when appropriate.]

(a) $\displaystyle\lim_{x \to 0} \frac{\ln(x+1)}{1 - e^{5x}}$

(b) $\displaystyle\lim_{x \to 0^+} \sqrt{x} \ln x$

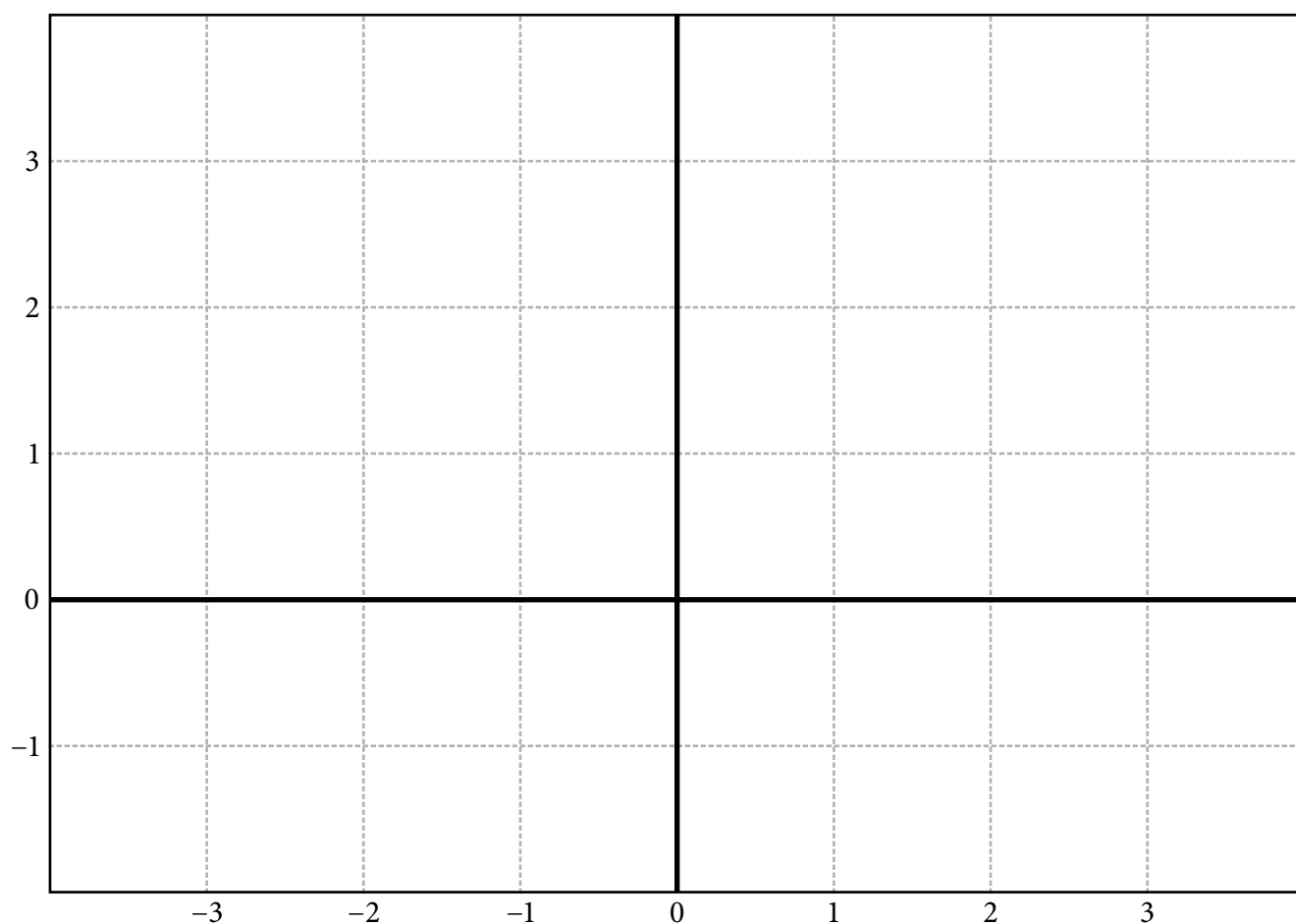(c) $\displaystyle\lim_{\theta \to \pi} \frac{\sin \theta}{1 - \cos \theta}$

3. (10 points) Show that the point on the curve $y = \frac{1}{x}$ with $x > 0$ that is closest to the point $(0,0)$ is the point $(1,1)$. For full credit, you must provide an argument showing that an absolute minimum is attained at the stated point.

4. (20 points) Sketch the graph of a function $f(x)$ that has the following properties.

- $f(x)$ is defined for all x except $x = -1$.
- $f(-3) = 1$, $f(-2) = 2$, $f(0) = 0$, and $f(2) = 2$.
- $f(x)$ has a vertical asymptote at $x = -1$.
- $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = 3$.
- $f'(x)$ is zero at $x = -2$ and $x = 0$, is positive for $x < -2$, $-2 < x < -1$ and $x > 0$, and is negative elsewhere.
- $f''(x)$ is zero at $x = -3$, $x = -2$ and $x = 2$, is positive for $x < -3$, $-2 < x < -1$ and $-1 < x < 2$, and is negative elsewhere.

On your graph, mark each point of inflection with a box, and classify all of the critical points.

5. (15 points)

    (a) Find the linearization of $f(x) = \sin(x)$ at $x = \pi/4$.

    (b) Use your linearization to approximate $\sin\left(\frac{\pi}{4} + \frac{1}{10}\right)$. Express your answer as a single fraction.
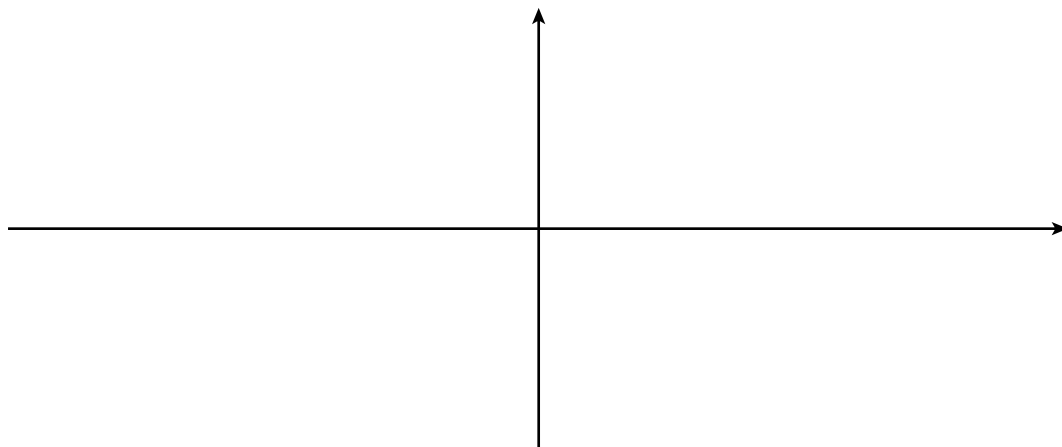
6. (15 points) Air is being slowly released from a spherical balloon. At time t = 0, the radius of the balloon is observed to be 10 cm and the radius is observed to be decreasing at the rate of 1 cm/s.

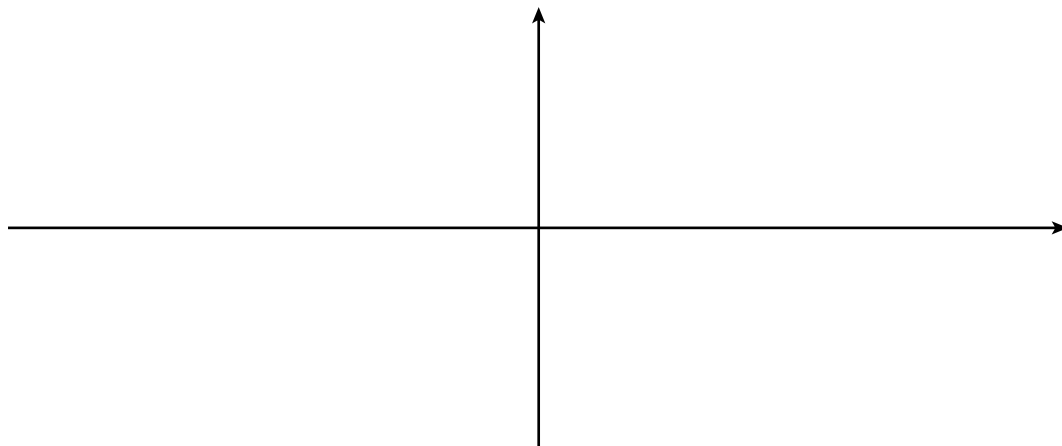   (a) Determine the rate of change of volume of the balloon when $r = 10$.

   (b) Assuming the rate of change of volume remains constant, how long will it take to empty the balloon?

7. (10 points) For each of the following scenarios, draw the graph of a function $f(x)$ with domain all of $\mathbb{R}$ that has a derivative at every point and that satisfies the desired criteria.

   (a) The function attains an absolute maximum value and has a local minimum, but does not attain an absolute minimum value.

   (b) The function has a critical point, but at no point has a local minimum or maximum value.

[**5 points extra credit:**] Formally state the Mean Value Theorem and use it to prove that for all real numbers a and b where $a < b$,

$$-(b - a) \le \sin b - \sin a \le b - a.$$