Math F251 Final Exam Spring 2020

Name: _____ **Section:** ☐ **F01 (Faudree)**
☐ **F02 (Bueler)**
☐ **UX1 (Van Spronsen)**

All students must affirm the following statements by initialing in the blanks provided. Students using their own paper must write out the statements in full.

_____ I will not seek or accept help from anyone.

_____ I will not use a calculator, books, notes, the internet or other aids.

_____ I understand that answers without work will not be awarded credit.

Good luck!

Problem	Possible	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

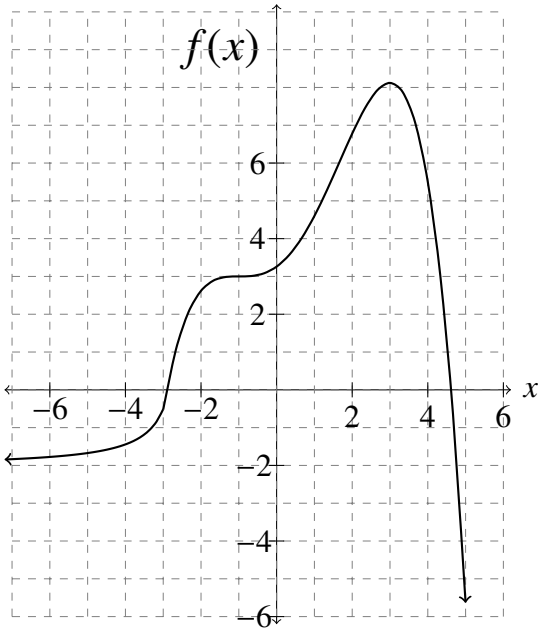
1. (10 points)

Sketch a graph $H(x)$ with all of the properties below. Label your graph.

- The domain of $H(x)$ is $(-\infty, 3) \cup (3, \infty)$.
- $H(0) = 1$
- $\lim_{x \to 0^-} H(x) = 2$
- $\lim_{x \to 0^+} H(x) = 0$
- $\lim_{x \to 3} H(x) = \infty$
- $H'(x) < 0$ and $H''(x) < 0$ on the interval $(-\infty, 0)$
- H has an inflection point when $x = 5$

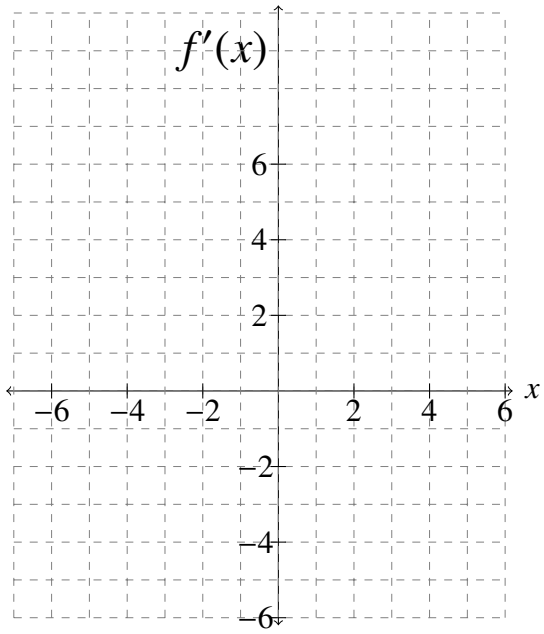
2. (10 points)

The graph of $f(x)$ is sketched below.



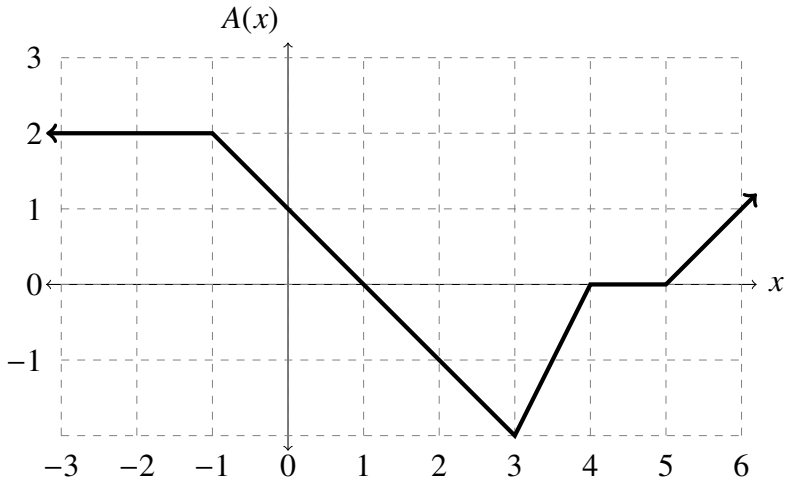
1. List the x-values of all critical numbers of f.

2. Use the graph of $f(x)$ to sketch the graph of $f'(x)$ on the set of axes below.



3. (10 points)

The function $A(x)$ is graphed below.



a. $A(0) =$

b. $A'(0) =$

c. At what x values, if any, does $A'(x)$ not exist?

d. By using your knowledge of areas, evaluate $\int_{-2}^{4} A(x)\,dx$.

For parts (e)-(g), let $H(x) = \int_{0}^{x} A(s)\,ds$.

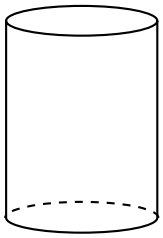
e. What is the value of $H(2)$?

f. What is the value of $H'(2)$?

g. Where on the interval $[0, 6]$ is $H(x)$ decreasing?

4. (10 points)

The height of a right circular cylinder is increasing at rate of 3 meters per second while its volume remains constant. (See figure below.) At what rate is the radius changing when the radius and height are both 10 meters?



5. (10 points)

Find any horizontal or vertical asymptotes for the function $f(x) = \frac{2x^2-3x}{5x^2-10}$. Use limits to justify your answer(s). If no asymptote exists, explain why.

6. (10 points)

A homeowner wants to minimize the cost of heating a building over the next 10 years. Adding x inches of insulation in the attic costs \$100 per inch and results in heating costs of $1000/(2 + x)$ dollars over 1 year. How many inches of insulation should be installed in order the minimize the total costs over a 10 year period? Justify your answer. (By **total costs**, we mean both the initial cost of insulating the building plus the annual heating costs.)

7. (10 points)

Evaluate the integrals below. Note that these problems will be graded **largely** by the quality of the work written. So make sure to include proper notation and compete steps.

a. $\displaystyle\int \sin(2x) + \frac{(1+\ln x)^2}{x} dx$

b. $\displaystyle\int_0^2 (1 + xe^{\pi x^2}) dx$

8. (10 points)

Use f, f' and f'' to answer the questions below.

$$f(x) = x\sqrt[3]{x^2-5} \qquad f'(x) = \frac{5(x^2-3)}{3(x^2-5)^{2/3}} \qquad f''(x) = \frac{10x(x^2-9)}{9(x^2-5)^{5/3}}.$$

a. Determine all critical numbers of the function f. Show how you obtain your answer.

b. For each critical number of f, classify it as a local minimum, a local maximum or neither. Show how you obtain your answer.

9. (10 points)

Short Answer

a. A population of chickadees is increasing at a rate of $r(t)$ chickadees per year. What does $\int_1^4 r(t)\,dt = 400$ mean? Make sure to include units in your answer.

b. Let $y = -3 + 5(x - 4)$ be an equation of the tangent line to the graph of $f(x)$ at $x = 4$. Is it possible to determine $f(4)$ or $f'(4)$? Explain your answer.

c. Let $C(T)$ be the number of chirps per second of a male cricket as a function of temperature, T, in degrees Fahrenheit. In the context of the problem, interpret $C'(70) = 2$. Make sure to include units in your answer.

10. (10 points)

The acceleration function (in m/s^2), the initial velocity, and the initial position are given for a particle moving along a line. Find an expression for position, s, at time t.

$$a(t) = \frac{12}{(1+t)^3}, \quad v(0) = 0, \quad s(0) = 0$$