Math F251   Midterm 1   Spring 2021

Name: ______________________

Student Id: ______________________

Calculator Model: ______________________

Section:
- ☐ FXA (Sus)
- ☐ FXB (Maxwell)
- ☐ UX1 (Jurkowski)

Rules:

You have 60 minutes to complete the exam.

Partial credit will be awarded, but you must show your work.

A scientific or graphing calculator (without symbolic manipulation) is allowed.

A one page sheet of paper (8 1/2 in. x 11 in.) with handwritten notes on one side is allowed.

No other aids are permitted.

Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.

If you need extra space, you can use the back sides of the pages. Please make it obvious when you have done so.

Turn off anything that might go beep during the exam.

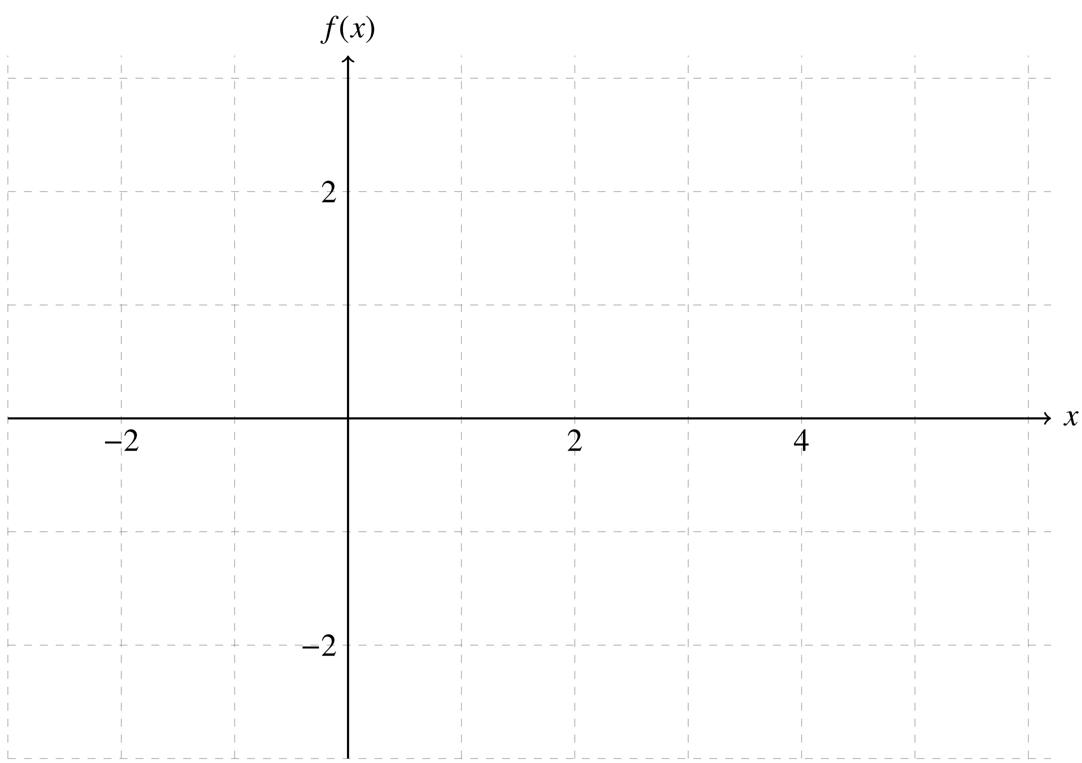
Good luck!

Problem	Possible	Score
1	9	
2	10	
3	16	
4	10	
5	10	
6	8	
Extra Credit	3	
Total	63	

1. (9 points)

In the diagram below, sketch the graph of a function $f(x)$ defined on all the real numbers satisfying the following criteria.

1. $\lim_{x\to 2} f(x) = 1$
2. $f(x)$ is not continuous at $x = 1$
3. $\lim_{x\to 4-} f(x) = \infty$
4. $f(4) = -1$
5. $\lim_{x\to\infty} f(x) = 0$
6. $\lim_{x\to 0^+} = -1$
7. $\lim_{x\to 0^-} = 2$
8. $f(-2) = -1$
9. $f(x)$ is a linear function on $[-2, 0]$



2. (10 points)

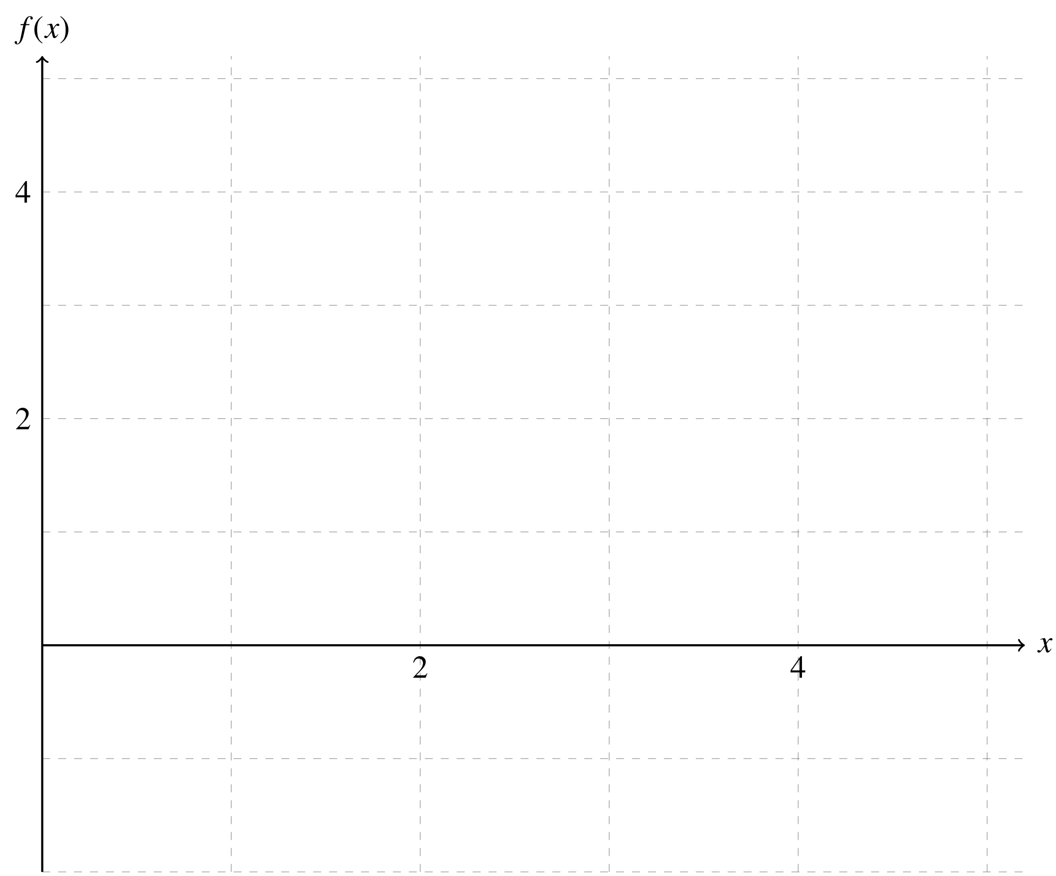
For a particular function $f(x)$,

$$f(1) = 4, \quad f(4) = 1, \quad f'(1) = 0 \quad \text{and} \quad f'(4) = 2.$$

a. Find the equation of the tangent line of the graph of $f(x)$ at $x = 4$.

b.

- On the axes below make a rough sketch of what the graph $y = f(x)$ might look like, using all of the limited information that you have.
- Sketch the tangent line at $x = 1$.
- Sketch the tangent line at $x = 4$.



3. (16 points)

Compute, with justification, the following limits, or explain why the given limit does not exist.

- **Use proper limit notation for full credit.**
- If a limit is **positive or negative infinity**, state that explicitly and justify your answer **without using a sketch**.

a. $\displaystyle\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h}$

Continued....

Problem 3 continued....

b. $\lim\limits_{x \to 5^+} \dfrac{x^2 - 3x}{5 - x}$

c. $\lim\limits_{x \to \infty} \dfrac{\sqrt{5x^2 - 2}}{3 - 2x}$

d. $\lim\limits_{x \to \infty} \sin\left(\dfrac{\sqrt{5x^2 - 2}}{3 - 2x}\right)$

4. (10 points)

An icicle is growing at the back of a house. At time $t \geq 0$ days the length of the icicle is

$$\ell(t) = \frac{14t + 4}{t + 2}$$

inches. Answer the following questions about the function $\ell(t)$ and be sure to include **units** in your answers.

a. Compute the average rate of change of the length of the icicle from time $t = 0$ to time $t = 2$ days. **Do not forget units!**

b. It is a known fact that $\ell'(t) = \dfrac{24}{(t+2)^2}$. Compute $\ell'(2)$ (**with units**) and explain what this quantity means about the icicle.

c. Compute $\lim_{t\to\infty} \ell(t)$. Then explain what this quantity means in precise but everyday language that the general public would understand. Do not forget units!

5. (10 points)

Use the **definition of the derivative** to compute $f'(2)$ if $f(x) = 7x^2$. No credit will be given if a different method is used.

6. (8 points)

Match the graph of each function (a) - (d) with the graph of its derivative I-IX. Write your answers in the blanks provided below.

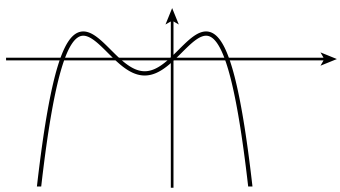
1. The derivative of graph (a) is ____________

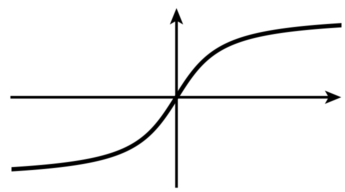
3. The derivative of graph (b) is ____________

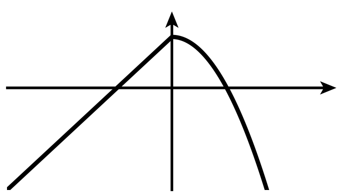
2. The derivative of graph (c) is ____________

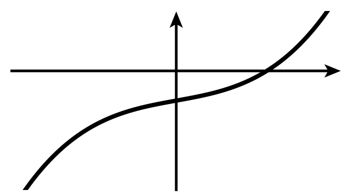
4. The derivative of graph (d) is ____________

Functions:

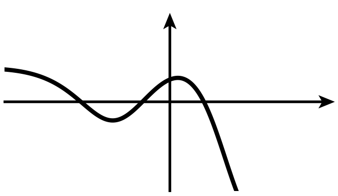
a) 

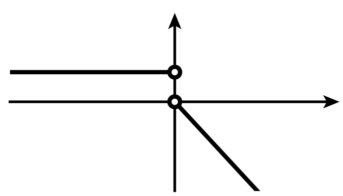
b) 

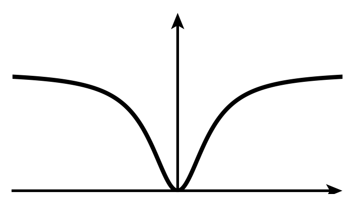
c) 

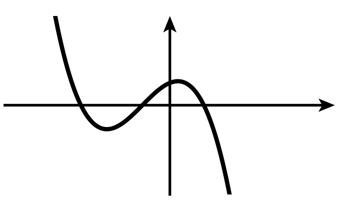
d) 

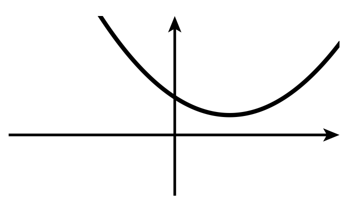
Derivatives:

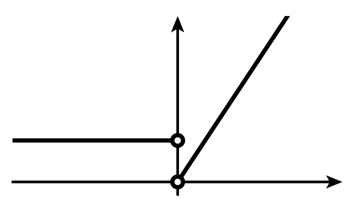
I. 

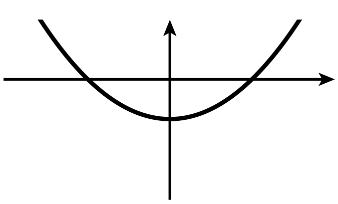
II. 

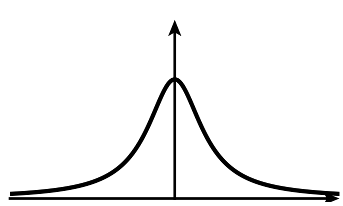
III. 

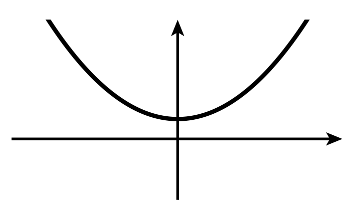
IV. 

V. 

VI. 

VII. 

VIII. 

IX. 

7. (Extra Credit: 3 points)

In problem **3a** you computed the following limit:

$$\lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h}.$$

In fact, this limit is a computation, from the definition of the derivative, of $f'(a)$ for some function $f(x)$ and some x-value a. What is $f(x)$ and what is a?