Math F251   Midterm 2   Spring 2023

Name: _____

Rules:

You have 90 minutes to complete the exam.

Partial credit will be awarded, but you must show your work.

You may have a single handwritten 3×5 notecard.
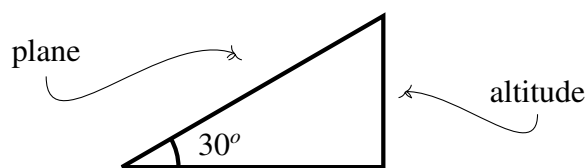
Calculators are not allowed.

Place a box around your FINAL ANSWER to each question where appropriate.

Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	10	
2	10	
3	12	
4	14	
5	16	
6	10	
7	8	
8	8	
9	12	
Extra Credit	5	
Total	100	

1. (10 points) If a airplane is flying at a constant speed of 100 miles per hour and is climbing at a an angle of 30 degrees, at what rate is its altitude changing? Your final answer should include units.

plane

altitude

30°

2. (10 points)

   (a) Find the linear approximation, $L(x)$, to $f(x) = \sqrt{x}$ at $a = 25$.

   (b) Use your answer in part a to estimate $\sqrt{23}$. Your answer should be in the form of a simplified fraction or a decimal.

3. (12 points) Evaluate the indefinite integrals below.

   (a) $\displaystyle\int \left(8x^{2/3} + \sec(x)\tan(x)\right)\, dx$

   (b) $\displaystyle\int (e^x + 2)\, dx$

   (c) $\displaystyle\int \frac{1 + x^3}{x^2}\, dx$

4. (14 points) Let $f(x) = \ln(x^2 + 2)$. It is a fact that $f'(x) = \dfrac{2x}{x^2 + 2}$ and $f''(x) = \dfrac{-2(x^2 - 2)}{(x^2 + 2)^2}$.

   (a) Determine intervals where $f(x)$ is increasing or decreasing.

   (b) Identify any local maxima or minima or state that none exist and their location.
      (Your answer should be in the form: "f has a local maximum/minimum of ____ at ____" or
      "f has no local maxima/minima.")

   (c) Determine intervals where $f(x)$ is concave up or concave down.

   (d) Identify all inflection points or state that none exist.

5. (16 points) Evaluate the limits below. You must show your work. Indicate an application of L'Hôpital's Rule by putting an H above equal sign.

(a) $\displaystyle\lim_{x\to 0}\frac{e^x - \cos(x)}{4\tan(x)}$

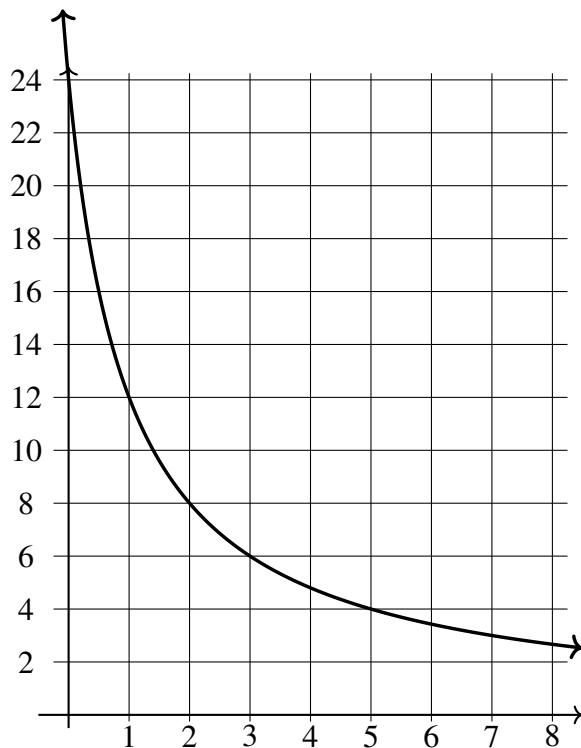(b) $\displaystyle\lim_{x\to\infty}\frac{\ln(x)}{x^{1/2}}$

(c) $\displaystyle\lim_{x\to 0^+}(1+x)^{\frac{1}{2x}}$

6. (10 points) An open-topped box with a square base has a fixed surface area of 1200 in^2. Determine the dimensions of the box with maximum volume. Justify your answer using Calculus.
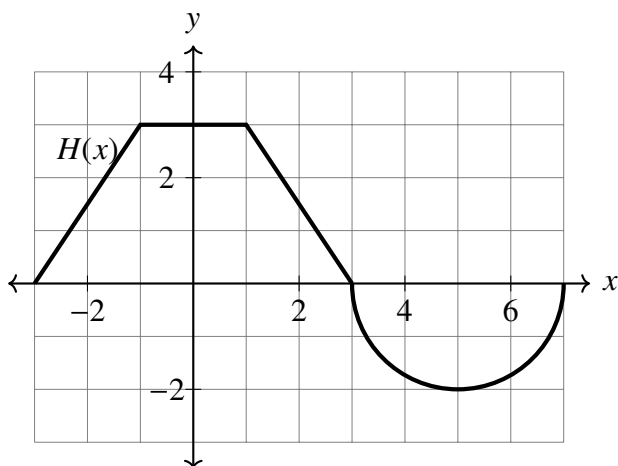
7. (8 points) The function $f(x) = \frac{24}{x+1}$ is graphed below. We want to estimate the area under the curve $f(x)$ on the interval $[0, 8]$ using M_4. (That is, we want to use 4 approximating rectangles with midpoints determining height.)

(a) Sketch the four approximating rectangles on the graph.

(b) Do a calculation to estimate the area under the curve using M_4 (that is, use 4 approximating rectangles and midpoints) and simplify your answer.

8. (8 points) Evaluate the definite integrals below using the graph of $H(x)$ and properties of definite integrals. On the interval $[3, 7]$, the graph of H is a semi-circle. Show your work.
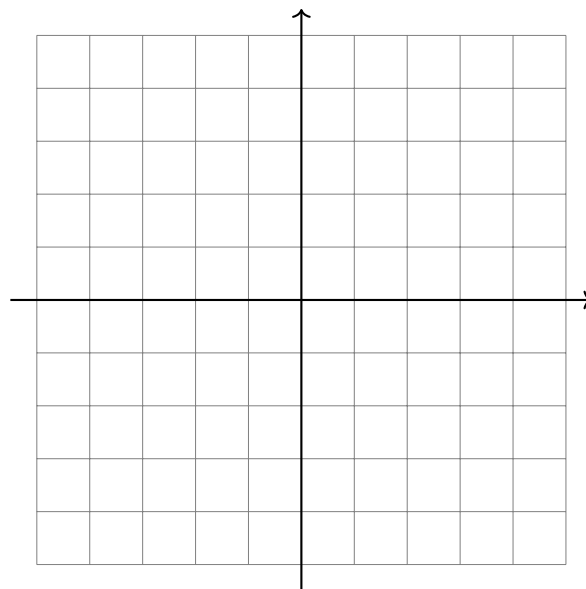

(a) $\int_{-3}^{7} f(x)\,dx$

(b) $\int_{0}^{3} (4f(x) + 6)\,dx$

9. (12 points) Sketch a graph that satisfies all the criteria in the list below.

- Domain $(-\infty, \infty)$

- $f(0) = 2$

- $f'(x) > 0$ on $(-\infty, \infty)$

- $f''(x) < 0$ on $(-\infty, 0)$, $f''(x) > 0$ on $(0, \infty)$

Extra Credit (5 points) Identify all vertical and horizontal asymptotes of the function $f(x) = \frac{4e^x+1}{7e^x-1}$. Justify your answer using limits.