Spring 2026 Math F251X

Calculus I: Final Exam

Name: ____________________

Section: ☐ 9:15 (James Gossell)
☐ 11:45 (Gordon Williams)
☐ Online (James Gossell)

Rules:

- Partial credit will be awarded, but you must **show your work**.
- You may have a single handwritten $3'' \times 5''$ notecard, both sides.
- Personal calculators are **not allowed**.
- Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.
- Turn off anything that might go beep during the exam.
- You have two hours to complete the exam.

Problem	Possible	Score
1	9	
2	9	
3	12	
4	12	
5	10	
6	10	
7	10	
8	14	
9	14	
Extra Credit	(5)	
Total	100	

1. (9 points)

Compute the following **limits**. Show your work clearly. Make sure you use **limit notation** where required and not where it isn't; an answer that does not use proper notation will not receive full credit. Use = to show things are equal. If you use L'Hôpital's rule, write $\overset{H}{=}$ or $\overset{L'H}{=}$ to indicate where you ~~are~~ applying it.

a. $\lim_{x\to\infty} \dfrac{3x^2 + 3e^{-2x}}{x^2 - e^{-x}} \overset{L'H}{=} \lim_{x\to\infty} \dfrac{6x - 6e^{-2x}}{2x + e^{-x}} \overset{L'H}{=} \lim_{x\to\infty} \dfrac{6 + 12\cancel{e^{-2x}}}{2 - \cancel{e^{-x}}} = 3$

type $\frac{\infty}{\infty}$ type $\frac{\infty}{\infty}$ $\to 0$

b. $\lim_{t\to 1^+} \dfrac{(t+1)(t+2)}{1 - t^2} = \lim_{t\to 1^+} \dfrac{(\cancel{t+1})(t+2)}{(\cancel{1+t})(1-t)} = -\infty$

$\to 3$

small and negative for $x>1$ as $x\to 1^+$

c. $\lim_{h\to 0} \dfrac{6xh - 3h^2 + h}{h} = \lim_{h\to 0} (6x - 3h + 1) = 6x+1$

2. (9 points)

Compute the following derivatives. Simplify your answers.

a. $\frac{d}{dx}\ln(3x)\sec(2x)$ $= \frac{1}{3x}\cdot x \cdot \sec(2x) + \ln(3x)\cdot \sec(2x)\tan(2x)\cdot 2$

$= \frac{1}{3}\sec(2x) + 2\ln(3x)\sec(2x)\tan(2x)$

$= \sec(2x)\left[\frac{1}{3} + 2\ln(3x)\tan(2x)\right]$

b. $\frac{d}{dt}\frac{t+1}{t-2}$ $= \frac{1(t-2)-(t+1)\cdot 1}{(t-2)^2} = \frac{t-2-t-1}{(t-2)^2} = \frac{-3}{(t-2)^2}$

c. $\frac{d}{dt}\sin(\cos^2(3t)+t)$ $= \cos(\cos^2(3t)+1)\cdot(2\cos(3t)\sin(3t)\cdot 3 + 1)$

$= \cos(\cos^2(3t)+1)\cdot(6\cos(3t)\sin(3t)+1)$

3. (12 points)

Compute the following **integrals**. Give the most general answer, and show your work. Clearly indicate any substitutions you use in such a way that someone else can follow your work. Do not put a $+C$ where it does not belong, and you must include $+C$ where it is needed.

a. $\displaystyle\int_1^3 10x^4 - 12x^2\,dx = 2x^5 - 4x^3 \Big|_1^3 = 2\cdot 3^5 - 4\cdot 3^3 - (2-4)$

$= 486 - 108 + 2 = 380$

b. $\displaystyle\int_{-2}^{2} \frac{2x+3}{(2x^2+6x)^{2/3}}\,dx = \int_{x=-2}^{x=2} \frac{2x+3}{u^{2/3}} \cdot \frac{du}{4x+6} = \frac{1}{2}\int_{x=-2}^{x=2} u^{-2/3}\,du$

$u = 2x^2 + 6x$

$\frac{du}{dx} = 4x + 6$

$\displaystyle = \frac{1}{2}\cdot\frac{3}{1}\cdot u^{1/3}\Big|_{x=-2}^{x=2} = \frac{3}{2}\left(2x^2+6x\right)^{1/3}\Big|_{x=-2}^{x=2}$

$\displaystyle = \frac{3}{2}\left((8+12)^{1/3} - (8-12)^{1/3}\right) = \frac{3}{2}\left(\sqrt[3]{20} - \sqrt[3]{-4}\right)$

$\displaystyle = \frac{3}{2}\left(\sqrt[3]{20} + \sqrt[3]{4}\right)$

c. $\displaystyle\int \frac{2x}{\sqrt{1-x^4}}\,dx = \int \frac{2x}{\sqrt{1-u^2}}\,\frac{du}{2x}$

$u = x^2$

$\frac{du}{dx} = 2x$

$\displaystyle = \int \frac{1}{\sqrt{1-u^2}}\,du = \arcsin(u) + C = \arcsin(x^2) + C.$

d. $\displaystyle\int x e^{3x^2}\,dx = \int x e^u \frac{du}{6x} = \frac{1}{6}e^u + C = \frac{1}{6}e^{3x^2} + C$

$u = 3x^2$

$\frac{du}{dx} = 6x$

4. (12 Points)

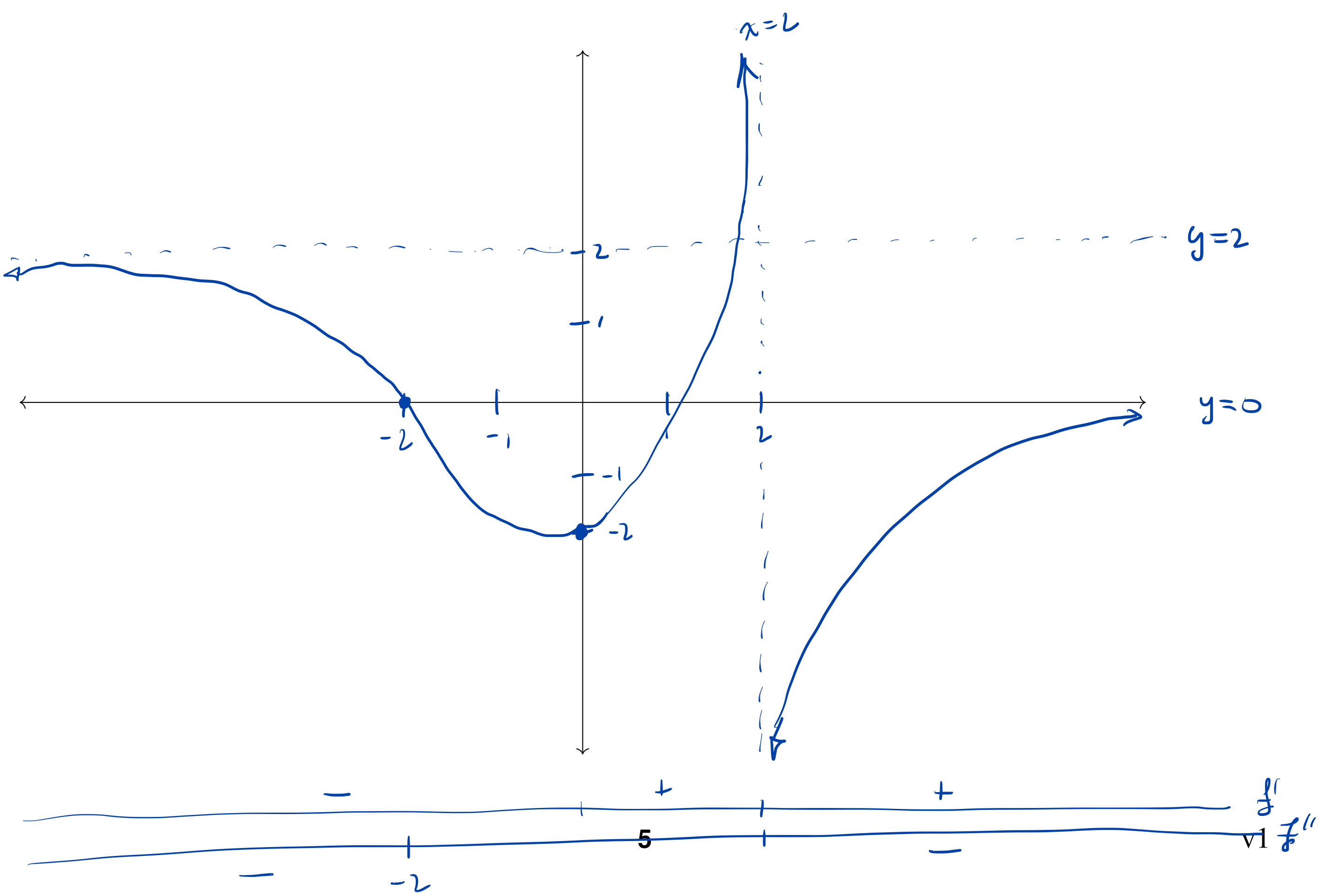
Sketch a graph of a function $f(x)$ that satisfies all of the following properties.

After drawing the graph:

- **Label** on the graph the following things, if they exist, by drawing a point on the graph and labeling: any local maximums by writing LOCAL MAX, local minimums by writing LOCAL MIN, inflection points by writing IP
- Draw any horizontal and vertical asymptotes with dashed lines and **label** them with their equation.
- Mark any important x-values and y-values on the x- and y-axes.

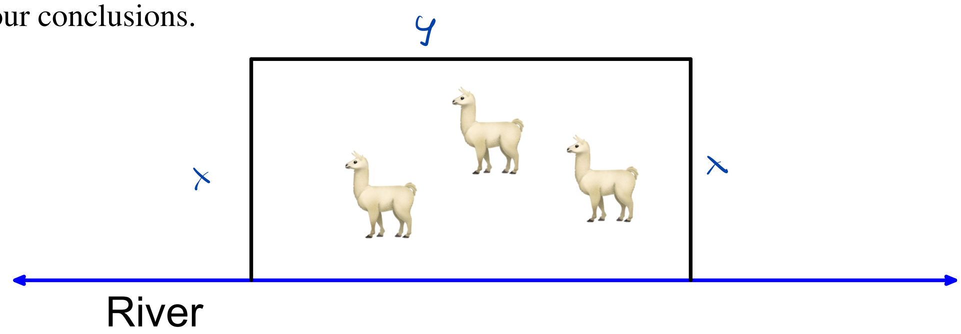
Properties:

- The domain of $f(x)$ is $(-\infty, 2) \cup (2, \infty)$
- $f(-2) = 0$
- $f(0) = -2$
- $\lim_{x\to\infty} f(x) = 0$
- $\lim_{x\to-\infty} f(x) = 2$
- $\lim_{x\to 2^+} f(x) = -\infty$
- $f'(x) > 0$ on $(0, 2) \cup (2, \infty)$
- $f'(x) < 0$ on $(-\infty, 0)$
- $f''(x) > 0$ on $(-2, 2)$
- $f''(x) < 0$ on $(\infty, -2) \cup (2, \infty)$



5. (10 Points)

You have 900 m of fencing to make a rectangular pen for llamas, with one side of the pen abutting a river (so no fence needed on that side). What are the dimensions of the pen that maximizes the enclosed area? **Include units.** Fully justify your conclusions.



$2x + y = 900\ m \Rightarrow y = 900 - 2x$

$x \cdot y = \text{Area} \Rightarrow x(900 - 2x) = A(x) = 900x - 2x^2$

$A'(x) = 900 - 4x \qquad A'(x) = 0$ when $x = \frac{900}{4} = 225\, m$

$A''(x) = -4$ so concave down on $(0, \infty)$

so $x = 225$ is location of a local maximum.

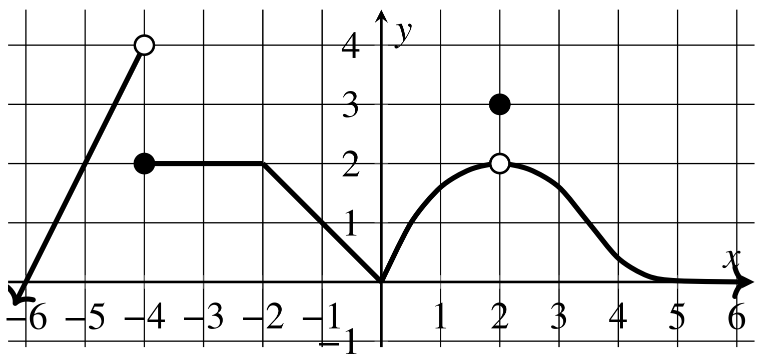
unique critical point on $(0, \infty)$ so global maximum.

Dimensions of pen: $y = 900 - 2 \cdot 225 = 450\, m$

$x = 225\, m.$

6. (10 points)

The graph of a function $g(x)$ is shown below. Use the graph to answer each question below. If the value does not exist or is undefined, write DNE.



a. $g'(-5) =$ 2

b. $\lim_{x\to -4^-} g(x) =$ 4

c. $\lim_{x\to -4} g(x) =$ 2

d. $g'(-3) =$ 0

e. $\int_{-4}^{0} g(x) =$ 6

f. $\lim_{x\to -2^+} g(x) =$ 2

g. $\lim_{x\to -2^+} g'(x) =$ −1

h. $\lim_{x\to 2} g(x) =$ 2

i. $\lim_{x\to \infty} g(x) =$ 0

j. List all **x-values** in the set $(-6, 6)$ where the function $g(x)$ is **not** continuous. Classify the discontinuity as **infinite**, **jump**, **removable**, or **other**.

$x =$ −4, jump. x = 2, removable

k. List all **x-values** in the set $(-6, 6)$ where the function $g(x)$ is **not** differentiable.

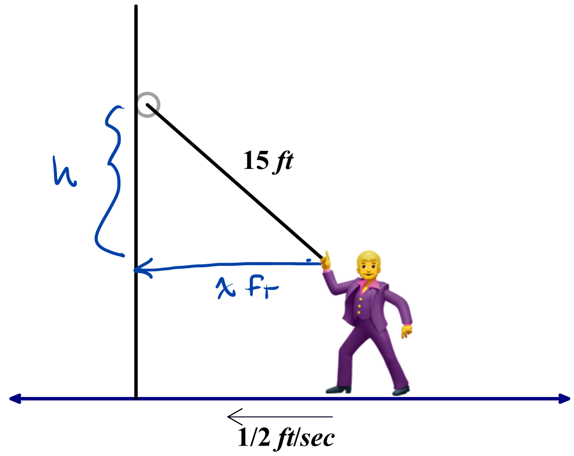
$x =$ −4, −2, 0, 2

l. Is $g''(1)$ positive, negative, or zero?

m. Is $g''(4)$ positive, negative, or zero?

7. (10 Points)

A painter is using a roller brush at the end of a 15 foot long pole to paint the wall of a building. Suppose the painter is moving towards the wall at a rate of $\frac{1}{2}$ ft/sec. When the painter is 9 feet from the wall, how fast is the brush head moving? Clearly identify whether the brush head is moving up or down. **Include units and simplify your answer**.



$$\frac{dx}{dt} = -2 \text{ ft/sec}$$

$$x^2 + h^2 = 15^2 \text{ ft}^2$$

$$2x\frac{dx}{dt} + 2h\frac{dh}{dt} = 0$$

$$\frac{dh}{dt} = -\frac{x}{h}\frac{dx}{dt}$$

when $x = 9$ ft, $h = \sqrt{15^2 - 9^2}$
$= 3\sqrt{5^2 - 3^2}$
$= 3 \cdot 4 = 12$ ft.

so $$\frac{dh}{dt} = -\frac{9 \text{ ft}}{12 \text{ ft}} \cdot \left(-\frac{1}{2} \text{ ft/sec}\right) = \frac{9}{24}\frac{\text{ft}}{\text{sec}}$$

$$= \frac{3}{8} \text{ ft/sec} > 0.$$

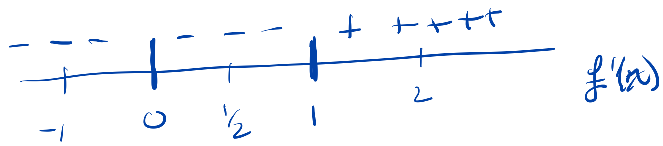
The brush is moving up the wall at a rate of $\frac{3}{8}$ ft/sec.

8. (14 Points)

Let $f(x) = 3x^4 - 4x^3$.

a. Find the locations of any local maxima or minima of $f(x)$ on $(-\infty, \infty)$.

$f'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$ crit. points at $x = 0, 1$.



local min. at $x=1$

no local max.

b. Find the absolute maximum and minimum of $f(x)$ on $[-1, 2]$.

By the Extreme value theorem, need to check values at $-1, 0, 1, 2$.

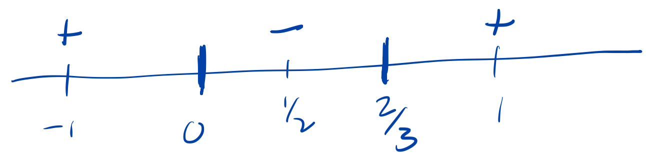
x	$f(x)$
-1	7
0	0
1	-1
2	$48 - 32 = 16$

abs. max is 16 at $x=2$

abs min is -1 at $x=1$

c. On what intervals is $f(x)$ concave up on $(-\infty, \infty)$? On what intervals is $f(x)$ concave down on $(-\infty, \infty)$? If no such an interval exists, say so, and justify your answer.

$f''(x) = 36x^2 - 24x = 12x(3x-2)$ $f''(x) = 0$ at $x = 0, \frac{2}{3}$.



concave up on $(-\infty, 0) \cup (2/3, \infty)$

concave down on $(0, 2/3)$

d. What are the locations (x-values) of any inflection points of $f(x)$ on $(-\infty, \infty)$? If there are none, say they do not exist and justify your conclusion.

Inflection points at 0 and 2/3 since $f''(x)$ changes sign.

9. (14 points)

A 4-story Foucault pendulum assumes small oscillations so that the motion is approximately simple harmonic. The **horizontal velocity** of the pendulum bob is given by:

$$v(t) = -\frac{9}{20}\sin\left(\frac{9}{10}t\right) \text{ meters per second}$$

At time $t = 0$ seconds, the pendulum is at its maximum displacement of 0.5 meters from equilibrium.

a. Find the horizontal position function $x(t)$ of the pendulum bob at t seconds.

$x(t) = \int -\frac{9}{20} \sin\left(\frac{9}{10}t\right) dt = \frac{1}{2}\cos\left(\frac{9}{10}t\right) + C$

$x(0) = 0.5$ so $C = 0$. So $x(t) = \frac{1}{2}\cos\left(\frac{9}{10}t\right)$ m.

b. Write a sentence interpreting the meaning of the value of $\int_0^5 v(t)\,dt$ in the context of the problem. **Include units**. **Do not solve**.

This is the net distance traveled by the bob in the first 5 seconds.

c. Find the acceleration function $a(t)$ of the pendulum bob at t seconds.

$a(t) = v'(t) = \frac{-81}{200}\cos\left(\frac{9}{10}t\right)$ m/s^2

d. Evaluate $a(1)$. You do not need to simplify your answer, but you do need to **include units**.

$a(1) = -\frac{81}{200}\cos(9/10)$ m/s^2

e. After 1 second, is the pendulum bob speeding up or slowing down? Explain your reasoning.

(Hint: sin(9/10) and cos(9/10) are both positive.)

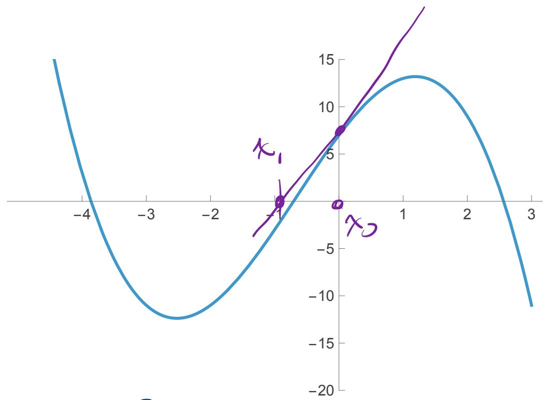
$v(t) < 0$ at $t = 1$, as is $a(t)$, so the bob is speeding up (in the negative direction)

10. (Extra Credit: 5 Points)

In this problem you will use Newton's method to estimate the value of a root of a function

$$f(x) = -x^3 - 2x^2 + 9x + 7,$$

whose graph is given at right.



a. Describe, in words, how Newton's method works; in other words, what is the main idea of Newton's method? Draw the first step on the graph showing how to obtain x_1 from $x_0 = 0$.

We draw the tangent line to the curve from a point near the root, and use the intersection of that line with the x-axis to determine x_1.

b. Will Newton's method obtain a good estimate for the **positive** root of $f(x)$ starting with $x_0 = 0$? Explain your answer.

No, the method will instead approach the root at about $x = -2/3$. We would need an x_0 closer to $x=2$.

c. Suppose I start my attempt to find the positive root of $f(x)$ with $x_0 = 2$. What is x_1? What is x_2?

$f'(x) = -3x^2 - 4x + 9$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 2 - (-9/11) = \dfrac{31}{11} \approx 2.8181\ldots$

$x_2 \approx 2.8181 - \dfrac{f(2.8181)}{f'(2.8181)} = 2.5920\ldots$