Spring 2026 **Math F251X**

Calculus I: Final Exam

Name: _____ **Section:** ☐ **9:15 (James Gossell)**
☐ **11:45 (Gordon Williams)**
☐ **Online (James Gossell)**

Rules:

- Partial credit will be awarded, but you must **show your work**.
- You may have a single handwritten $3'' \times 5''$ notecard, both sides.
- Personal calculators are **not allowed**.
- Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.
- Turn off anything that might go beep during the exam.
- You have two hours to complete the exam.

Problem	Possible	Score
1	9	
2	9	
3	12	
4	12	
5	10	
6	10	
7	10	
8	14	
9	14	
Extra Credit	(5)	
Total	100	

1. (9 points)

Compute the following **limits**. Show your work clearly. Make sure you use **limit notation** where required and not where it isn't; an answer that does not use proper notation will not receive full credit. Use = to show things are equal. If you use L'Hôpital's rule, write $\overset{H}{=}$ or $\overset{L'H}{=}$ to indicate where you are applying it.

a. $\lim_{x\to\infty} \dfrac{3x^2 + 3e^{-2x}}{x^2 - e^{-x}}$

b. $\lim_{t\to 1^+} \dfrac{(t+1)(t+2)}{1-t^2}$

c. $\lim_{h\to 0} \dfrac{6xh - 3h^2 + h}{h}$

2. (9 points)

Compute the following derivatives. Simplify your answers.

a. $\dfrac{d}{dx}\ln(3x)\sec(2x)$

b. $\dfrac{d}{dt}\dfrac{t+1}{t-2}$

c. $\dfrac{d}{dt}\sin(\cos^2(3t)+t)$

3. (12 points)

Compute the following **integrals**. Give the most general answer, and show your work. Clearly indicate any substitutions you use in such a way that someone else can follow your work. Do not put a $+C$ where it does not belong, and you must include $+C$ where it is needed.

a. $\displaystyle\int_1^3 10x^4 - 12x^2\,dx$

b. $\displaystyle\int_{-2}^2 \frac{2x+3}{(2x^2+6x)^{2/3}}\,dx$

c. $\displaystyle\int \frac{2x}{\sqrt{1-x^4}}\,dx$

d. $\displaystyle\int xe^{3x^2}\,dx$

4. (12 Points)

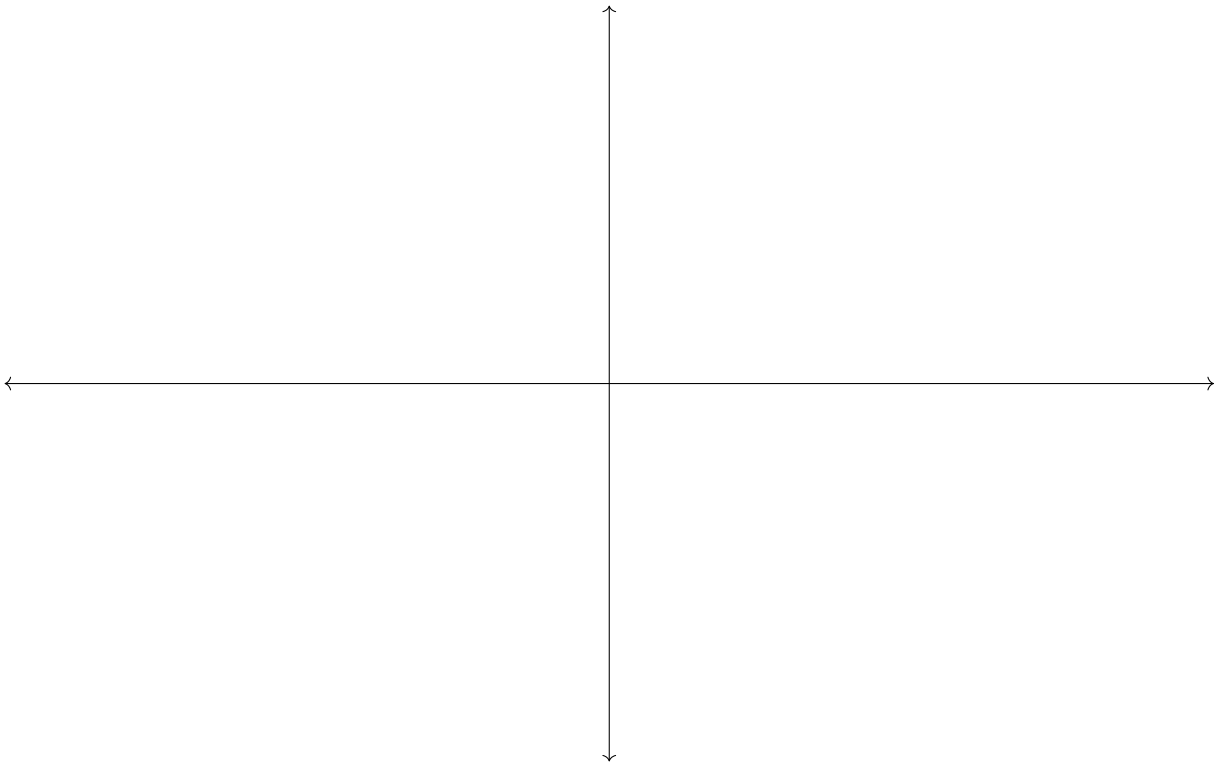
Sketch a graph of a function $f(x)$ that satisfies all of the following properties.

After drawing the graph:

- **Label** on the graph the following things, if they exist, by drawing a point on the graph and labeling: any local maximums by writing LOCAL MAX, local minimums by writing LOCAL MIN, inflection points by writing IP
- Draw any horizontal and vertical asymptotes with dashed lines and **label** them with their equation.
- Mark any important x-values and y-values on the x- and y-axes.

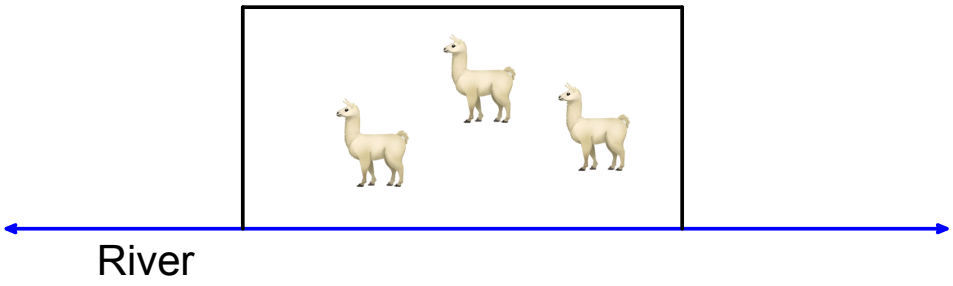
Properties:

- The domain of $f(x)$ is $(-\infty, 2) \cup (2, \infty)$
- $f(-2) = 0$
- $f(0) = -2$
- $\lim_{x \to \infty} f(x) = 0$
- $\lim_{x \to -\infty} f(x) = 2$
- $\lim_{x \to 2^+} f(x) = -\infty$
- $f'(x) > 0$ on $(0, 2) \cup (2, \infty)$
- $f'(x) < 0$ on $(-\infty, 0)$
- $f''(x) > 0$ on $(-2, 2)$
- $f''(x) < 0$ on $(\infty, -2) \cup (2, \infty)$



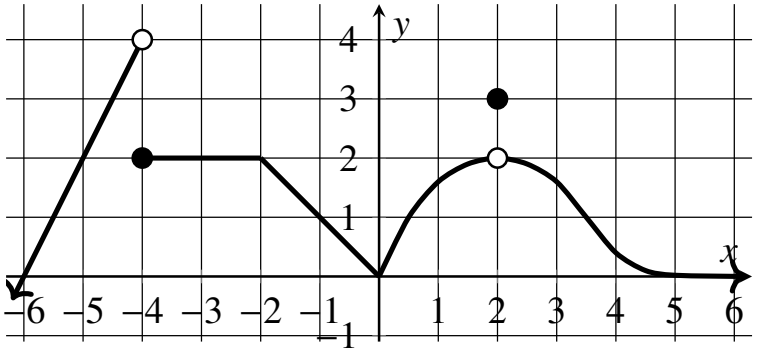
5. (10 Points)

You have 900 m of fencing to make a rectangular pen for llamas, with one side of the pen abutting a river (so no fence needed on that side). What are the dimensions of the pen that maximizes the enclosed area? **Include units.** Fully justify your conclusions.



6. (10 points)

The graph of a function $g(x)$ is shown below. Use the graph to answer each question below. If the value does not exist or is undefined, write DNE.



a. $g'(-5) =$ _____

b. $\lim_{x \to -4^-} g(x) =$ _____

c. $\lim_{x \to -4} g(x) =$ _____

d. $g'(-3) =$ _____

e. $\int_{-4}^{0} g(x) =$ _____

f. $\lim_{x \to -2^+} g(x) =$ _____

g. $\lim_{x \to -2^+} g'(x) =$ _____

h. $\lim_{x \to 2} g(x) =$ _____

i. $\lim_{x \to \infty} g(x) =$ _____

j. List all **x-values** in the set $(-6, 6)$ where the function $g(x)$ is **not** continuous. Classify the discontinuity as **infinite**, **jump**, **removable**, or **other**.

$x =$ _____

k. List all **x-values** in the set $(-6, 6)$ where the function $g(x)$ is **not** differentiable.

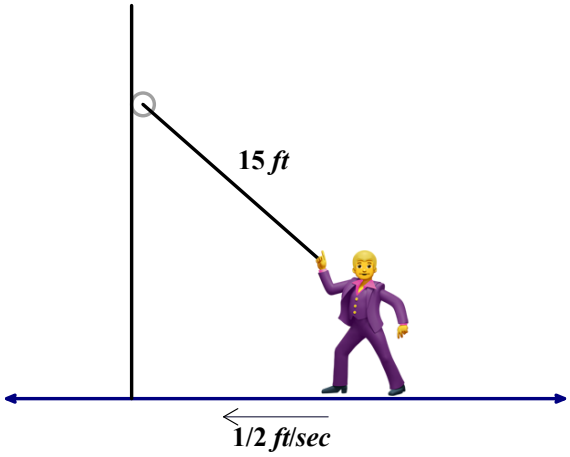
$x =$ _____

l. Is $g''(1)$ positive, negative, or zero?

m. Is $g''(4)$ positive, negative, or zero?

7. (10 Points)

A painter is using a roller brush at the end of a 15 foot long pole to paint the wall of a building. Suppose the painter is moving towards the wall at a rate of $\frac{1}{2}$ ft/sec. When the painter is 9 feet from the wall, how fast is the brush head moving? Clearly identify whether the brush head is moving up or down. **Include units and simplify your answer**.



8. (14 Points)

Let $f(x) = 3x^4 - 4x^3$.

a. Find the locations of any local maxima or minima of $f(x)$ on $(-\infty, \infty)$.

b. Find the absolute maximum and minimum of $f(x)$ on $[-1, 2]$.

c. On what intervals is $f(x)$ concave up on $(-\infty, \infty)$? On what intervals is $f(x)$ concave down on $(-\infty, \infty)$? If no such an interval exists, say so, and justify your answer.

d. What are the locations (x-values) of any inflection points of $f(x)$ on $(-\infty, \infty)$? If there are none, say they do not exist and justify your conclusion.

9. (14 points)

A 4-story Foucault pendulum assumes small oscillations so that the motion is approximately simple harmonic. The **horizontal velocity** of the pendulum bob is given by:

$$v(t) = -\frac{9}{20}\sin\left(\frac{9}{10}t\right) \text{ meters per second}$$

At time $t = 0$ seconds, the pendulum is at its maximum displacement of 0.5 meters from equilibrium.

a. Find the horizontal position function $x(t)$ of the pendulum bob at t seconds.

b. Write a sentence interpreting the meaning of the value of $\int_0^5 v(t)\,dt$ in the context of the problem. **Include units**. **Do not solve**.

c. Find the acceleration function $a(t)$ of the pendulum bob at t seconds.

d. Evaluate $a(1)$. You do not need to simplify your answer, but you do need to **include units**.

e. After 1 second, is the pendulum bob speeding up or slowing down? Explain your reasoning.

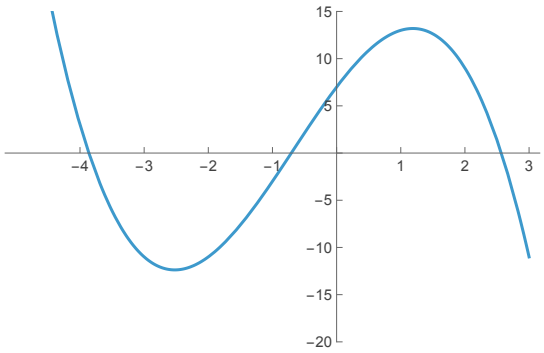
(Hint: $\sin(9/10)$ and $\cos(9/10)$ are both positive.)

10. (Extra Credit: 5 Points)

In this problem you will use Newton's method to estimate the value of a root of a function

$$f(x) = -x^3 - 2x^2 + 9x + 7,$$

whose graph is given at right.



a. Describe, in words, how Newton's method works; in other words, what is the main idea of Newton's method? Draw the first step on the graph showing how to obtain x_1 from $x_0 = 0$.

b. Will Newton's method obtain a good estimate for the **positive** root of $f(x)$ starting with $x_0 = 0$? Explain your answer.

c. Suppose I start my attempt to find the positive root of $f(x)$ with $x_0 = 2$. What is x_1? What is x_2?

$x_1 =$

$x_2 =$