Spring 2026 Math F251X

Calculus 1: Midterm 1

Name: ______________________

Section:
- ☐ **9:15am (James Gossell)**
- ☐ **11:45am (Gordon Williams)**
- ☐ **async (James Gossell)**

Rules:

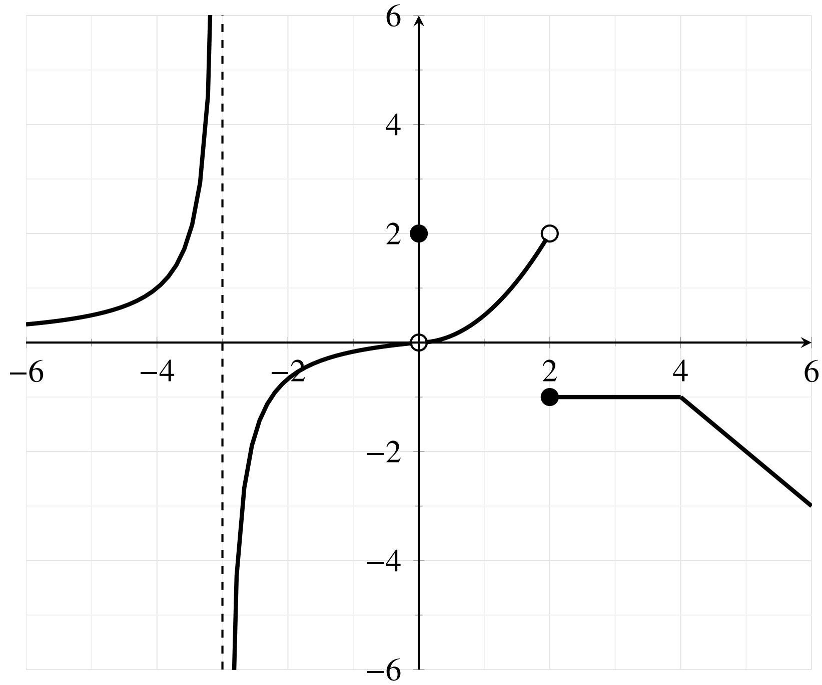
- You will have 90 minutes to take this exam.
- Partial credit will be awarded, but you must show your work.
- You may have a single handwritten $3'' \times 5''$ notecard, both sides.
- Calculators are **not** allowed.
- Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.
- Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	16	
2	8	
3	16	
4	10	
5	10	
6	12	
7	16	
8	12	
Extra Credit	(5)	
Total	100	

1. (16 points)

Use the graph of $f(x)$ to answer the following questions.



a. Fill in the blanks below. If the value does not exist or is undefined, write DNE.

$\lim_{x\to -3^-} f(x) =$ +∞ $\qquad \lim_{x\to 0} f(x) =$ 0 $\qquad f(0) =$ 2

$\lim_{x\to 2^-} f(x) =$ 2 $\qquad \lim_{x\to 2^+} f(x) =$ -1 $\qquad \lim_{x\to 2} f(x) =$ DNE

$f'(3) =$ 0 $\qquad \lim_{x\to 4^-} f'(x) =$ 0 $\qquad \lim_{x\to -4^+} f'(x) =$ 1

b. State the x-values for which f is **not continuous**. For each of your answers, classify the discontinuity as **jump**, **removable**, **infinite**, or **other**.

-3, infinite 0, removable 2, jump

c. State the x-values for which f is **not differentiable** (where $f'(x)$ does not exist).

-3, 0, 2 and 4

2. (8 points)

The distance d in centimeters of a ball from a wall at time t is given by $d(t) = 10 + 3\cos\left(\frac{5\pi}{3}t\right)$, where t is measured in seconds. We are interested in the velocity of the ball after one second.

In the table below, v_{avg} denotes the average velocity of the ball on the time interval between the time 1 second, and the time t seconds. Note that $d(1) = 23/2$.

t sec	$d(t)$ cm	v_{avg} cm/sec
0.00000	13	$-3/2$
0.9	10.	15.
0.99	11.362	13.8029
0.999	11.4864	13.624
0.9999	11.4986	13.6056
0.99999	11.4999	13.6037
1.00001	11.5001	13.6033
1.0001	11.5014	13.6014
1.001	11.5136	13.5829
1.01	11.6339	13.3917
1.1	12.5981	10.9808
2	17/2	-3

$d(0) = 10 + 3\cos(0) = 13$ see

$V_{avg} = \dfrac{23/2 - 13}{1 - 0} = \dfrac{-3/2}{1} = -3/2$ cm/sec

a. Fill in the missing entries in the table above (for $t = 0.00000$ seconds).

b. Using the data provided in the table above, estimate a value for the instantaneous velocity of the ball when $t = 1$ second.

$v_{inst} \approx 13.6035$ cm/sec

c. Is the ball moving towards the wall, or away from the wall when $t = 1$ second?

away, since $v_{inst} > 0$.

3. (16 points)

Evaluate the following limits. Show your work. Use limit notation where necessary; you will be graded both on your computation and on your correct use of notation. If the limit does not exist, write DNE and a few words about why it does not exist. If the limit is unbounded, write ∞ or $-\infty$ as appropriate, and again, justify your answer with a few words.

a. $\lim\limits_{x\to-3} \dfrac{\sqrt{x+4}-1}{x+3} = \cancel{\dfrac{\sqrt{-3+4}-1}{-3+3} = 0/0}$

$= \lim\limits_{x\to-3} \dfrac{\sqrt{x+4}-1}{x+3} \cdot \dfrac{\sqrt{x+4}+1}{\sqrt{x+4}+1} = \lim\limits_{x\to-3} \dfrac{(x+4)-1}{(x+3)(\sqrt{x+4}+1)}$.

$= \lim\limits_{x\to-3} \dfrac{x+3}{(x+3)(\sqrt{x+4}+1)} = \lim\limits_{x\to-3} \dfrac{1}{\sqrt{x+4}+1} = \dfrac{1}{2}$.

b. $\lim\limits_{t\to2} \dfrac{\frac{1}{2}-\frac{1}{t}}{t-2} = \cancel{0/0}$

$= \lim\limits_{t\to2} \dfrac{1}{t-2} \cdot \left(\dfrac{t}{2t} - \dfrac{2}{2t}\right) = \lim\limits_{t\to2} \dfrac{t-2}{2t(t-2)} = \lim\limits_{t\to2} \dfrac{1}{2t} = \dfrac{1}{4}$

c. $\lim\limits_{\theta\to0} \dfrac{2\theta}{\cos(\theta)} = \dfrac{2\cdot0}{1} = 0$

d. $\lim\limits_{x\to1^+} \dfrac{2xe^{-x}}{x^2(1-x)}$ Numerator goes to $2e^{-1}$, denominator is negative, and goes to zero. Hence,

$\lim\limits_{x\to1^+} \dfrac{2xe^{-x}}{x^2(1-x)} = -\infty$.

4. (10 points)

Consider the function

$$f(x) = x^2 + 3x.$$

Find $f'(-2)$ using the **limit definition of the derivative** and show your work using all appropriate notation. **No credit will be awarded for using other methods**. Begin by writing down the limit definition of the derivative. You must write limits where necessary to receive full credit.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{1}{h}\left[(x+h)^2 + 3(x+h) - [x^2+3x]\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h - \cancel{x^2} - \cancel{3x}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[2xh + h^2 + 3h\right] = \lim_{h\to 0}(2x + h + 3)$$

$$= 2x + 3.$$

So $f'(-2) = -4 + 3 = -1.$

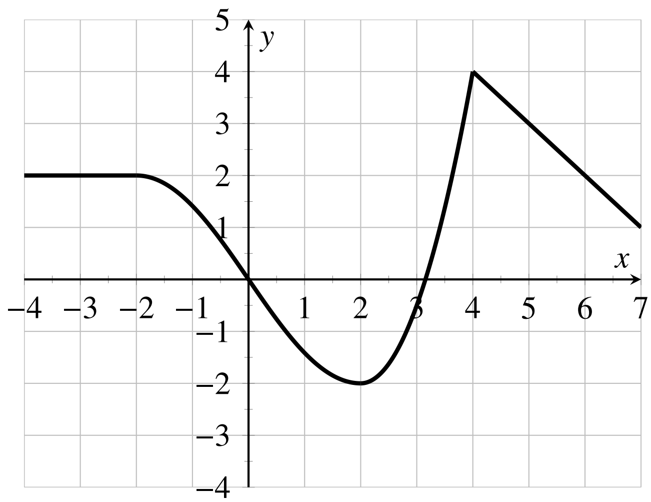
OR $$f'(-2) = \lim_{h\to 0} \frac{f(-2+h) - f(-2)}{h} = \lim_{h\to 0} \frac{(-2+h)^2 + 3(-2+h) - ((-2)^2 + 3(-2))}{h}$$

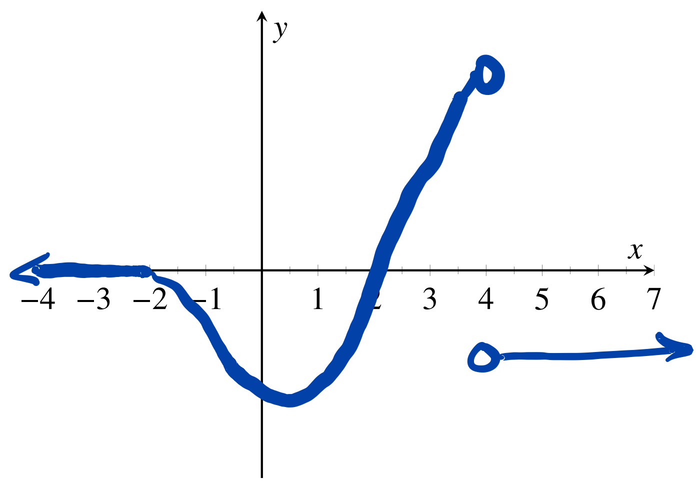
$$= \lim_{h\to 0} \frac{\cancel{4} - 4h + h^2 - \cancel{6} + 3h - \cancel{4} + \cancel{6}}{h}$$

$$= \lim_{h\to 0} \frac{h^2 - h}{h} = \lim_{h\to 0}(h-1) = -1.$$

5. (10 points)

The graph of $f(x)$ is shown below. On the **other set of axes**, sketch the graph of $f'(x)$. If there are any asymptotes, draw them with dashed lines. Use open circles to show points where the derivative is not defined, if any. (You are not given values on the y-axis; I am interested in the correct shape/holes/asymptotes of the derivative, not the specific values.)





6. (12 points)

Find the derivatives of the following functions. Use appropriate derivative notation (like "$f'(x) =$").

a. $f(x) = x^4 + 4x + \dfrac{1}{x^4} + \cos\left(\frac{\pi}{4}\right)$

$f'(x) = 4x^3 + 4 - 4x^{-5}$

b. $g(x) = \dfrac{x^2\sin(x)}{2x-1}$ P g

$$g'(x) = \frac{P'g - Pg'}{g^2} = \frac{(2x\sin(x) + x^2\cos(x))\cdot(2x-1) - 2\cdot x^2\sin(x)}{(2x-1)^2}$$

c. $h(x) = \sqrt{x}(x^2 - 7x + 12)$ $= x^{5/2} - 7x^{3/2} + 12x^{1/2}$

$h'(x) = \frac{5}{2}x^{3/2} - \frac{21}{2}x^{1/2} + 6x^{-1/2}$

7. (16 points)

A drone is launched from the ground. Its **height** (in feet) at time t seconds is given by the function $h(t) = \dfrac{32t}{t+1}$.

a. What is the **initial velocity** of the drone?

$$h'(t) = \frac{32(t+1) - 32t}{(t+1)^2} = v(t)$$

$h'(0) =$ initial velocity $= \dfrac{32-0}{1^2} = 32$ ft/sec. $= v(0)$

b. After 3 seconds, what is the **velocity** of the drone?

$$h'(3) = \frac{32(4) - 32\cdot 3}{4^2} = \frac{32}{16} = 2\text{ ft/sec} = v(3)$$

c. Someone has determined that the acceleration of the drone at time t seconds is given by

$$a(t) = -\frac{64}{(t+1)^3}.$$

Describe in a sentence how they obtained the acceleration from $h(t)$.

they took the derivative with respect to t twice.

d. Is the drone speeding up or slowing down when $t = 3$ seconds? Explain your reasoning.

$a(3) = \dfrac{-64}{(3+1)^3} < 0$, while $v(3) = 2 > 0$, so

slowing down.

8. (12 points)

A machine extrudes a metal onto a conveyor belt. Let $M(l)$ denote the amount of mass measured in kilograms extruded after the machine has travelled l meters along the belt.

a. What does $\dfrac{M(b) - M(a)}{b - a}$ for $0 < a < b$ measure, and what are the units?

It measures the average mass per meter deposited between position a and b on the conveyor. The units are $\frac{kg}{m}$.

b. What does $M'(l)$ measure, and what are the units?

This is the instantaneous rate of change of mass with respect to the distance traveled. It is also measured in kg/m.

c. Suppose that $M(4) = 152$, and $M'(4) = 13$. Estimate $M(5)$. Be sure to include appropriate units in your answer.

$M(5) \approx M(4) + M'(4) \cdot 1m = 152\,kg + 13\,kg/m \cdot m = 165\,kg.$

d. Can $M'(l)$ be negative? Explain your reasoning.

No. That would mean the extruder was taking mass of the conveyor belt as it traveled.

9. (Extra Credit: 5 points)

Suppose that

$$g(x) = \begin{cases} 4x - x^2 & x \le 3 \\ 9 - 2x & x > 3 \end{cases}.$$

Is the $g(x)$ differentiable at $x = 3$? (In other words, does $g'(3)$ exist?) If so, what is $g'(3)$? Either way, justify your conclusions.

$g(3) = 12 - 9 = 3$, $\lim_{x \to 3^+} g(x) = 9 - 6 = 3.$

$\lim_{x \to 3^-} g(x) = 12 - 9 = 3$

So continuous at $x = 3$.

$\lim_{x \to 3^-} g'(x) = \lim_{x \to 3^-} (4 - 2x) = 4 - 6 = -2$

$\lim_{x \to 3^+} g'(x) = \lim_{x \to 3^+} (-2) = -2.$

So yes, $g'(3) = -2$.