Spring 2026 **Math F251X**

Calculus 1: Midterm 1

Name: _____ **Section:** ☐ **9:15am (James Gossell)**
☐ **11:45am (Gordon Williams)**
☐ **async (James Gossell)**

Rules:

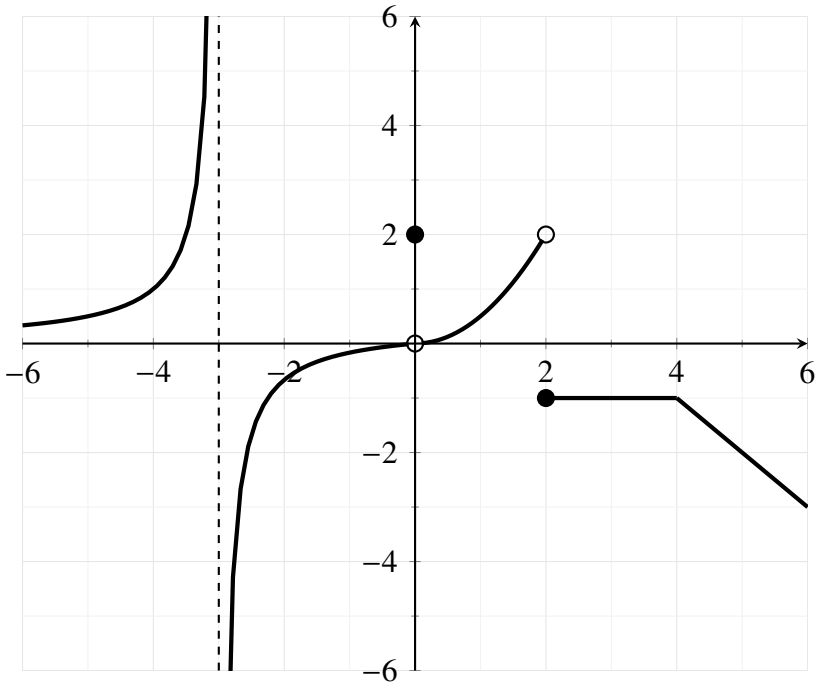
- You will have 90 minutes to take this exam.
- Partial credit will be awarded, but you must show your work.
- You may have a single handwritten $3'' \times 5''$ notecard, both sides.
- Calculators are **not** allowed.
- Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.
- Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	16	
2	8	
3	16	
4	10	
5	10	
6	12	
7	16	
8	12	
Extra Credit	(5)	
Total	100	

1. (16 points)

Use the graph of $f(x)$ to answer the following questions.



a. Fill in the blanks below. If the value does not exist or is undefined, write DNE.

$\lim_{x\to -3^-} f(x) =$ _____ $\lim_{x\to 0} f(x) =$ _____ $f(0) =$ _____

$\lim_{x\to 2^-} f(x) =$ _____ $\lim_{x\to 2^+} f(x) =$_____ $\lim_{x\to 2} f(x) =$ _____

$f'(3) =$ _____ $\lim_{x\to 4^-} f'(x) =$_____ $\lim_{x\to -4^+} f'(x) =$ _____

b. State the x-values for which f is **not continuous**. For each of your answers, classify the discontinuity as **jump**, **removable**, **infinite**, or **other**.

c. State the x-values for which f is **not differentiable** (where $f'(x)$ does not exist).

2. (8 points)

The distance d in centimeters of a ball from a wall at time t is given by $d(t) = 10 + 3\cos\left(\frac{5\pi}{3}t\right)$, where t is measured in seconds. We are interested in the velocity of the ball after one second.

In the table below, v_{avg} denotes the average velocity of the ball on the time interval between the time 1 second, and the time t seconds. Note that $d(1) = 23/2$.

t sec	$d(t)$ cm	v_{avg} cm/sec
0.00000		
0.9	10.	15.
0.99	11.362	13.8029
0.999	11.4864	13.624
0.9999	11.4986	13.6056
0.99999	11.4999	13.6037
1.00001	11.5001	13.6033
1.0001	11.5014	13.6014
1.001	11.5136	13.5829
1.01	11.6339	13.3917
1.1	12.5981	10.9808
2	17/2	-3

a. Fill in the missing entries in the table above (for $t = 0.00000$ seconds).

b. Using the data provided in the table above, estimate a value for the instantaneous velocity of the ball when $t = 1$ second.

c. Is the ball moving towards the wall, or away from the wall when $t = 1$ second?

3. (16 points)

Evaluate the following limits. Show your work. Use limit notation where necessary; you will be graded both on your computation and on your correct use of notation. If the limit does not exist, write DNE and a few words about why it does not exist. If the limit is unbounded, write ∞ or $-\infty$ as appropriate, and again, justify your answer with a few words.

a. $\lim_{x\to -3} \dfrac{\sqrt{x+4}-1}{x+3}$

b. $\lim_{t\to 2} \dfrac{\frac{1}{2}-\frac{1}{t}}{t-2}$

c. $\lim_{\theta\to 0} \dfrac{2\theta}{\cos(\theta)}$

d. $\lim_{x\to 1^+} \dfrac{2xe^{-x}}{x^2(1-x)}$

4. (10 points)

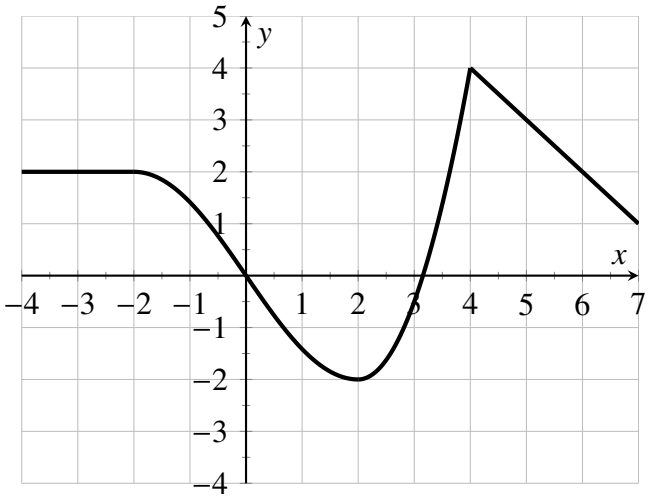
Consider the function

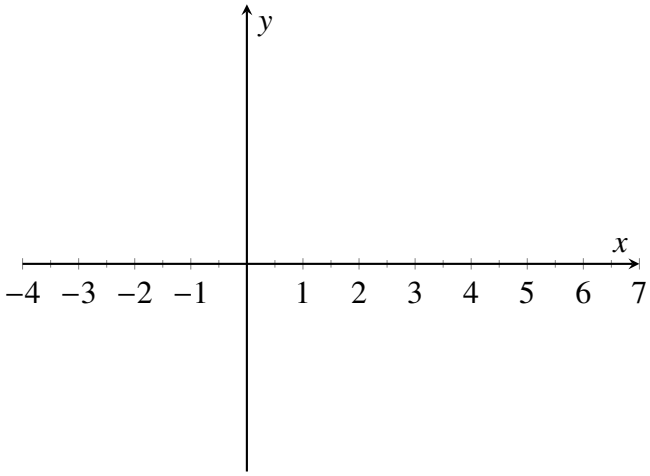
$$f(x) = x^2 + 3x.$$

Find $f'(-2)$ using the **limit definition of the derivative** and show your work using all appropriate notation. **No credit will be awarded for using other methods**. Begin by writing down the limit definition of the derivative. You must write limits where necessary to receive full credit.

5. (10 points)

The graph of $f(x)$ is shown below. On the **other set of axes**, sketch the graph of $f'(x)$. If there are any asymptotes, draw them with dashed lines. Use open circles to show points where the derivative is not defined, if any. (You are not given values on the y-axis; I am interested in the correct shape/holes/asymptotes of the derivative, not the specific values.)





6. (12 points)

Find the derivatives of the following functions. Use appropriate derivative notation (like "$f'(x) =$").

a. $f(x) = x^4 + 4x + \dfrac{1}{x^4} + \cos\left(\frac{\pi}{4}\right)$

b. $g(x) = \dfrac{x^2 \sin(x)}{2x - 1}$

c. $h(x) = \sqrt{x}(x^2 - 7x + 12)$

7. (16 points)

A drone is launched from the ground. Its **height** (in feet) at time t seconds is given by the function $h(t) = \dfrac{32t}{t+1}$.

a. What is the **initial velocity** of the drone?

b. After 3 seconds, what is the **velocity** of the drone?

c. Someone has determined that the acceleration of the drone at time t seconds is given by

$$a(t) = -\frac{64}{(t+1)^3}.$$

Describe in a sentence how they obtained the acceleration from $h(t)$.

d. Is the drone speeding up or slowing down when $t = 3$ seconds? Explain your reasoning.

8. (12 points)

A machine extrudes a metal onto a conveyor belt. Let $M(l)$ denote the amount of mass measured in kilograms extruded after the machine has travelled l meters along the belt.

a. What does $\dfrac{M(b) - M(a)}{b - a}$ for $0 < a < b$ measure, and what are the units?

b. What does $M'(l)$ measure, and what are the units?

c. Suppose that $M(4) = 152$, and $M'(4) = 13$. Estimate $M(5)$. Be sure to include appropriate units in your answer.

d. Can $M'(l)$ be negative? Explain your reasoning.

9. (Extra Credit: 5 points)

Suppose that

$$g(x) = \begin{cases} 4x - x^2 & x \leq 3 \\ 9 - 2x & x > 3 \end{cases}.$$

Is the $g(x)$ differentiable at $x = 3$? (In other words, does $g'(3)$ exist?) If so, what is $g'(3)$? Either way, justify your conclusions.