Spring 2026 **Math F251X**

Calculus 1: Midterm 2

Name: ______________________ **Section:** ☐ **9:15am (James Gossell)**
☐ **11:45am (Gordon Williams)**
☐ **async (James Gossell)**

Rules:

- Partial credit will be awarded, but you must show your work.
- You may have a single handwritten $3'' \times 5''$ notecard, both sides.
- Calculators are **not** allowed.
- Place a box around your $\boxed{\text{FINAL ANSWER}}$ to each question where appropriate.
- Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	12	
2	6	
3	10	
4	16	
5	16	
6	12	
7	12	
8	8	
9	8	
Extra Credit	(6)	
Total	100	

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1. (12 points)

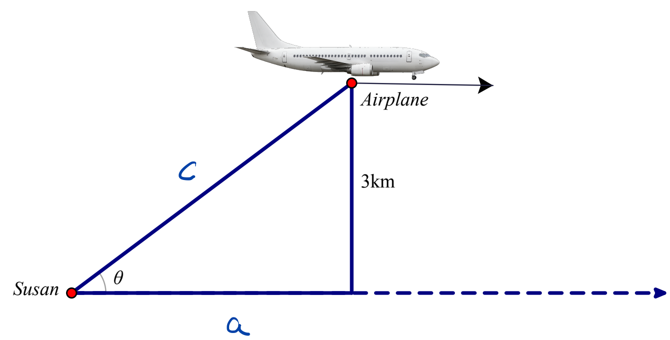
Susan is using an infrared laser to measure the distance to an airplane that is flying away from her at a speed of 400 km/hr. The plane is flying at an elevation of 3km. At the instant the plane is flying over a point 4 km away from Susan, at what speed must Susan be adjusting the **angle of elevation** θ the laser is pointing?

Show your work and include units in your final answer. You do not need to simplify your final answer.

Know:

$$\frac{da}{dt} = 400 \text{ km/hr.}$$

Want: when $a = 4$ km what is $\frac{d\theta}{dt}$?



Note: $\tan(\theta) = \frac{3\text{km}}{a\text{ km}} = \frac{3}{a}$

so $\sec^2(\theta) \cdot \frac{d\theta}{dt} = -3a^{-2}\frac{da}{dt}$ or $\frac{d\theta}{dt} = \frac{\cos^2(\theta)\cdot(-3)}{a^2}\frac{da}{dt}$

Note: $\cos(\theta) = \frac{a}{c}$. When $a = 4$ km, $c = 5$ km.

Thus when $a = 4$km, $\frac{d\theta}{dt} = \frac{16\text{km}^2}{25\text{km}^2} \cdot \frac{-3\text{km}}{16\text{km}^2} \cdot \frac{400\text{ km}}{\text{hr}}$

$$= -\frac{3}{25} \cdot 400 \frac{\text{radians}}{\text{hr}}$$

$$= -48 \frac{\text{radians}}{\text{hr.}}$$

2. (6 points)

Suppose $f(x) = 2\sqrt[3]{x}$.

a. Find the linear approximation $L(x)$ to $f(x)$ near $a = 64$.

$f'(x) = \frac{2}{3} x^{-2/3}$ $\qquad$ $f'(64) = \frac{2}{3} \frac{1}{64^{2/3}} = \frac{2}{3} \cdot \frac{1}{4^2} = \frac{1}{24}$

$L(x) = \frac{1}{24}(x-64) + 2\sqrt[3]{64} = \frac{1}{24}(x-64) + 8$

b. Use your answer in part (a) to estimate $\sqrt[3]{538} = 2\sqrt[3]{66}$. Simplify your answer.

$\sqrt[3]{538} \approx L(66) = \frac{1}{24}(66-64) + 8 = \frac{2}{24} + 8 = 8\frac{1}{12} = \frac{97}{12}$

3. (10 points)

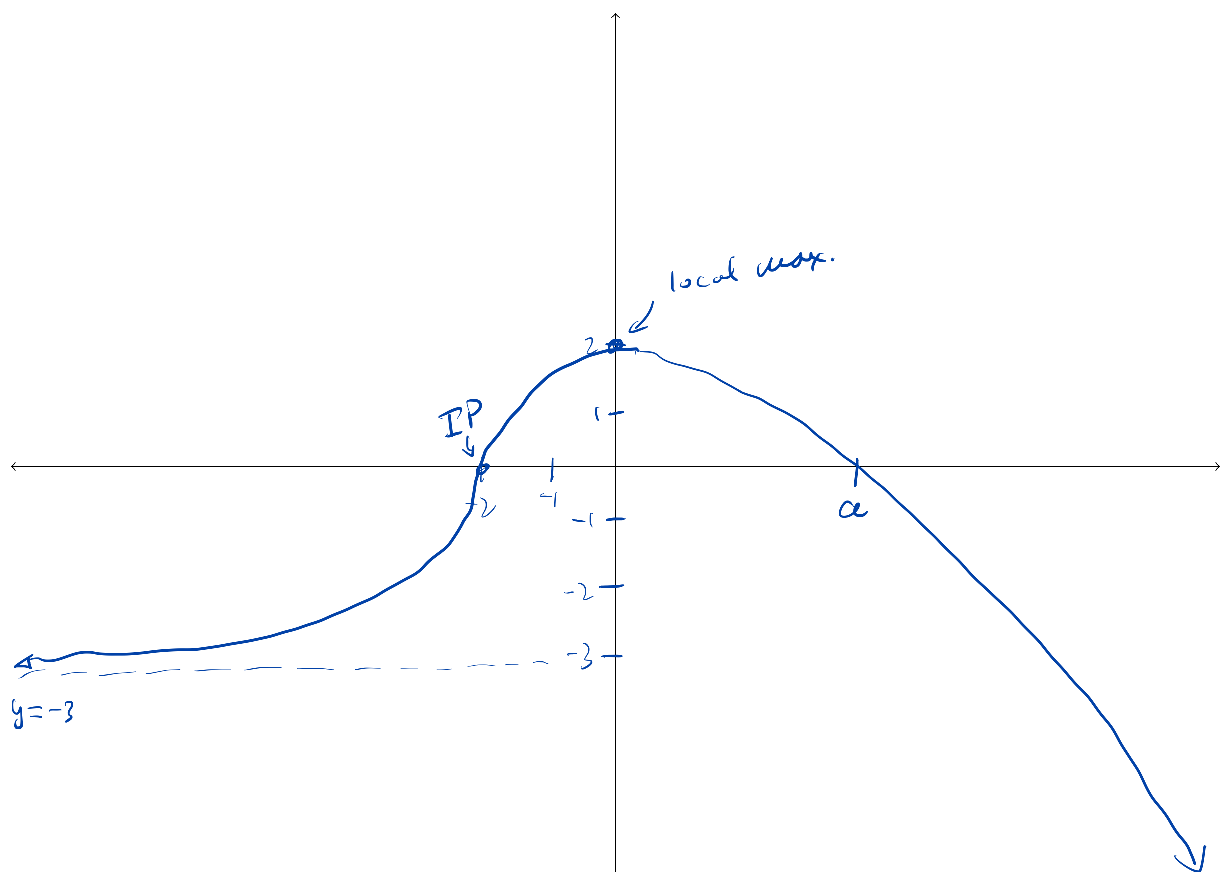
Sketch a graph of a function $f(x)$ that satisfies all of the following properties.

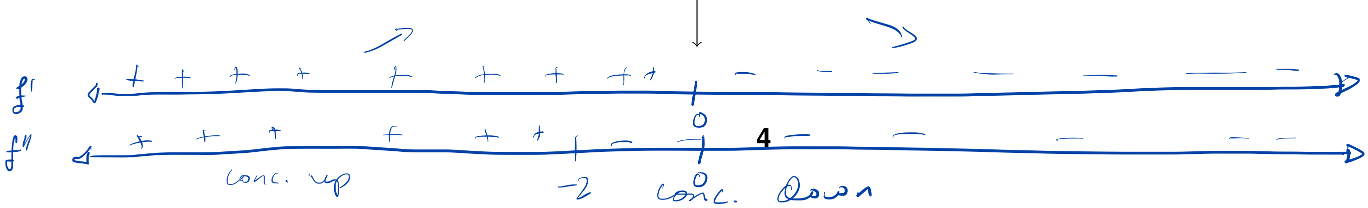
After drawing the graph:

- **Label** on the graph the following things, if they exist, by drawing a point on the graph and labeling: any local maximums by writing LOCAL MAX, local minimums by writing LOCAL MIN, inflection points by writing IP
- Draw any horizontal and vertical asymptotes with dashed lines and **label them with their equation**.
- Mark any important x-values and y-values on the x- and y-axes.

Properties:

- The domain of $f(x)$ is $(-\infty, \infty)$
- $f(-2) = 0$
- $f(0) = 2$
- $\lim_{x\to-\infty} f(x) = -3$
- $f'(x) > 0$ on $(-\infty, 0)$
- $f'(x) < 0$ on $(0, \infty)$
- $f''(x) > 0$ on $(-\infty, -2)$
- $f''(x) < 0$ on $(-2, \infty)$





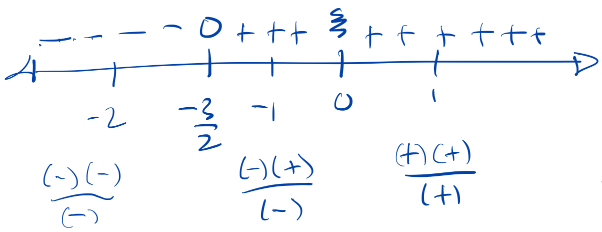
4. (16 points)

Let $f(x) = \dfrac{x^2(7+3x)}{\sqrt[5]{x}}$. It is a fact that $f'(x) = \dfrac{21x(3+2x)}{5\sqrt[5]{x}}$ and $f''(x) = \dfrac{126(2+3x)}{25\sqrt[5]{x}}$. Your answers to the questions below about $f(x)$ must show the work you used to draw your conclusions.

a. Determine the intervals where $f(x)$ is increasing or decreasing. Identify the locations of any local maxima or minima and their locations, or state that none exist.

$f'(x) = 0 = \dfrac{21x(3+2x)}{5\sqrt[5]{x}}$ or when $3+2x=0$, i.e., $x = -\frac{3}{2}$.

Also, $f(x)$ and $f'(x)$ undefined at $x=0$.



increasing on $(-\frac{3}{2}, 0) \cup (0, \infty)$

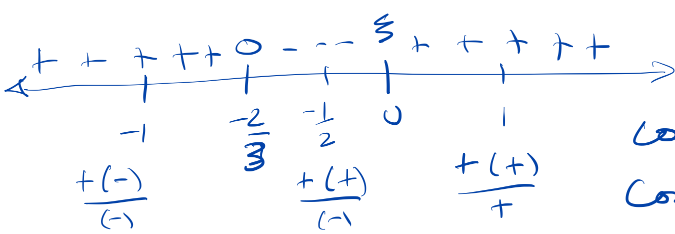
decreasing on $(-\infty, 0)$

By 1st derivative test, local minimum at $x=0$.

No local maxima.

b. Determine intervals where $f(x)$ is concave up or concave down.

$f''(x) = \dfrac{126(2+3x)}{25\sqrt[5]{x}}$ undef. at $x=0$, $f''(x)=0$ when $2+3x=0$, i.e., $x = -\frac{2}{3}$.



concave up on $(-\infty, -2/3) \cup (0, \infty)$

concave down on $(-2/3, 0)$

c. Identify the locations of all inflection points, or state that none exist.

There is an inflection point at $x = \frac{-2}{3}$.

5. (16 points)

Evaluate the following limits. Show your work, including appropriate use of limit notation. If you use L'Hôpital's rule, you must indicate where you are using it by writing $\overset{H}{=}$ or $\overset{L'H}{=}$, or similar, and you must indicate why you are using it (i.e., what is the form/type of limit?). Use ∞ or $-\infty$ where appropriate, and if the limit does not exist, write DNE and provide justification.

a. $\lim\limits_{t\to -2} \dfrac{e^{2t+4} - 2t + 3}{t^2 - 4t + 4} = \dfrac{e^{2(-2)+4} - 2(-2) + 3}{(-2)^2 - 4(-2) + 4} = \dfrac{1 + 4 + 3}{4 + 8 + 4} = \dfrac{8}{16} = \dfrac{1}{2}$

b. $\lim\limits_{t\to \infty} \dfrac{t^9 - 9t}{t^2 - 9t^9} = \lim\limits_{t\to\infty} \dfrac{1 - 9t^{-8}}{t^{-7} - 9} = \dfrac{1}{-9}$

c. $\lim\limits_{t\to 0} \dfrac{4\sin(t) - 4t}{e^{-t} + t - 1} \overset{L'H}{=} \lim\limits_{t\to 0} \dfrac{4\cos(t) - 4}{-e^{-t} + 1} \overset{L'H}{=} \lim\limits_{t\to 0} \dfrac{-4\sin(t)}{e^{-t}} = \dfrac{0}{1} = 0$

type $\frac{0}{0}$ type $\frac{0}{0}$

d. $\lim\limits_{t\to 1} \csc(\pi t)\ln(t) = \lim\limits_{t\to 1} \dfrac{\ln(t)}{\sin(\pi t)} \overset{L'H}{=} \lim\limits_{t\to 1} \dfrac{1/t}{\pi\cos(\pi t)} = \dfrac{1}{\pi(-1)} = \dfrac{-1}{\pi}$

type $\frac{0}{0}$

6. (12 points)

Determine which point (x, y) on the line given by $y = 3 - 2x$ is **closest** to the origin $(0, 0)$. Use calculus to justify your answer.

Note that the distance between two points (x_1, y_1) and (x_2, y_2) is given by $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Let d denote the squared distance between $(x, 3-2x)$ on the line, and $(0,0)$. Thus

$$d(x) = (x-0)^2 + ((3-2x)-0)^2 = x^2 + (3-2x)^2$$
$$= x^2 + 9 - 12x + 4x^2 = 5x^2 - 12x + 9.$$

(Minimizing D is the same as minimizing $D^2 = d$.)

$$\frac{d}{dx} d(x) = \frac{d}{dx}(5x^2 - 12x + 9) = 10x - 12.$$

Solve: $10x - 12 = 0$, get $x = \frac{6}{5}$.

$$\frac{d^2}{dx^2} d(x) = \frac{d}{dx}(10x - 12) = 10 > 0$$ so minimum.

at $\left(\frac{6}{5}, 3 - 2\cdot\left(\frac{6}{5}\right)\right) = \left(\frac{6}{5}, \frac{3}{5}\right)$

Final Answer: $(x, y) = (\frac{6}{5}, \frac{3}{5})$

7. (12 points)

Compute the following **antiderivatives** (indefinite integrals). Give the most general answer, and show your work using correct notation.

a. $\int \sec^2(x) + \sqrt[3]{2x^5} + 3e^x \, dx$ $= \int \sec^2(x) + \sqrt[3]{2}\, x^{5/3} + 3e^x \, dx$

$= \tan(x) + \frac{5\sqrt[3]{2}}{3} x^{2/3} + 3e^x + C$

b. $\int x^3(4x^2 - 2x^4)\, dx$ $= \int 4x^5 - 2x^7 \, dx$

$= \frac{4}{6}x^6 - \frac{2}{8}x^8 + C = \frac{2}{3}x^6 - \frac{1}{4}x^8 + C$

c. $\int \frac{2x^2 - x + 2}{x^2}\, dx$ $= \int 2 - \frac{1}{x} + 2x^{-2} \, dx$

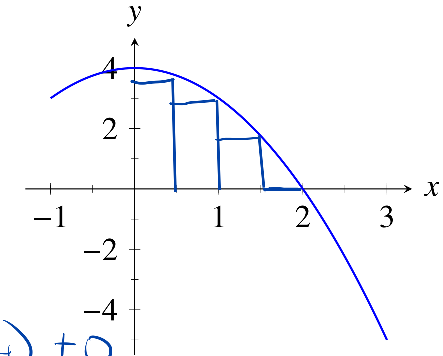
$= 2x - \ln|x| - 2x^{-1} + C$

$= 2x - \ln|x| - \frac{2}{x} + C$

8. (8 points)

Each of the problems below are about estimating areas associated with the curve $y = -x^2 + 4$.

a. Draw the rectangles needed to estimate the integral $\int_0^2 -x^2 + 4\,dx$ using the right hand endpoint rule with $n = 4$ rectangles.
What value does this estimate provide for the integral?

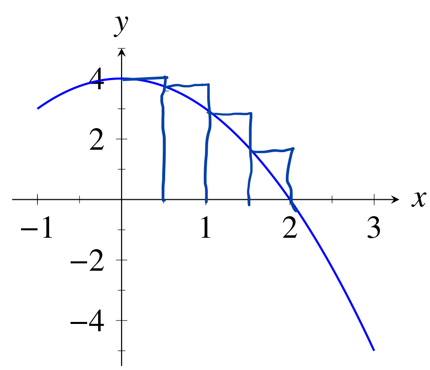


$$\text{Area} \approx \frac{1}{2}\left(-\left(\tfrac{1}{2}\right)^2+4\right) + \frac{1}{2}\left(-(1)^2+4\right) + \frac{1}{2}\left(-\left(\tfrac{3}{2}\right)^2+4\right) + 0$$

$$= \frac{1}{2}\left[4 - \frac{1}{4} + 4 - 1 + 4 - \frac{9}{4}\right]$$

$$= \frac{1}{2}\left[12 - \frac{14}{4}\right] = \frac{1}{2}\left[\frac{48-14}{4}\right] = \frac{34}{8} = \frac{17}{4} \text{ units}^2.$$

b. Draw the rectangles needed to estimate the integral $\int_0^2 -x^2 + 4\,dx$ using the left hand endpoint rule with $n = 4$ rectangles. (Do not evaluate.)



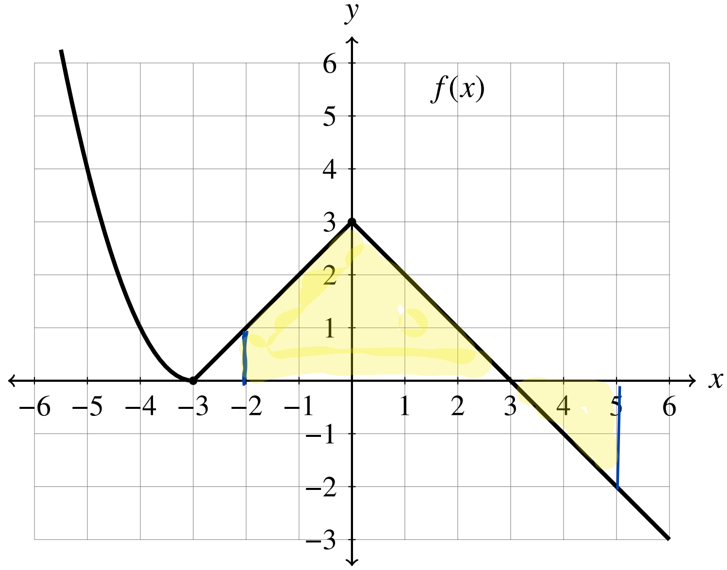
c. Which of these will be an overestimate? An underestimate? Briefly explain your reasoning.

The left hand rule will be an over estimate, because the rectangles cover the area under the curve, plus some more.

the right hand rule will be an underestimate since the rectangles leave some area under the curve uncovered.

9. (8 points)

Consider the graph of the function $f(x)$ shown below:



a. Determine $\int_{-2}^{5} f(x)\,dx$. Show some work or say something about what you computed and how.

$\int_{-2}^{5} f(x)\,dx = 4 + \frac{9}{2} - 2 = \frac{13}{2}$.

I added up the squares and half squares in the shaded region, keeping track of which regions were above and below the x-axis.

b. Determine $\int_{-2}^{5} -2f(x) + 3\,dx$. (Hint: Use part (a) above.)

$\int_{-2}^{5} -2f(x) + 3\,dx = -2 \cdot \int_{-2}^{5} f(x)\,dx + \int_{-2}^{5} 3\,dx$

$= -2 \cdot \left(\frac{13}{2}\right) + 3 \cdot 7 = -13 + 21 = 8$

10. (Extra Credit: 6 points)

a. Compute the sum $\sum_{i=1}^{5} 4i^2$.

$$\sum_{i=1}^{5} 4i^2 = 4\cdot1^2 + 4\cdot2^2 + 4\cdot3^2 + 4\cdot4^2 + 4\cdot5^2$$
$$= 4(1+4+9+16+25) = 4(55) = 220$$

b. Write, using sigma notation, the sum

$$\frac{1}{3} + \frac{4}{3} + 3 + \frac{16}{3} + \frac{25}{3} + 12.$$

$$\sum_{i=1}^{6} \frac{i^2}{3}$$

c. Write the sum needed to estimate $\int_1^3 5x^2\,dx$, using the left hand endpoint method with n equal width rectangles (in the notation of the book, this would be L_n). To get full credit you will need to use sigma notation.

$$L_n = \sum_{i=0}^{n-1} 5\left(1+\frac{2i}{n}\right)^2 \cdot \frac{2}{n}$$