Intro Video:
Derivatives involving logarithmic and exponential functions

Math F251X Calculus 1

Example 1: $\int e^{3x}\,dx$

Use substitution! $u = 3x \Rightarrow \frac{du}{dx} = 3 \Rightarrow dx = \frac{du}{3}$

Then $\int e^{3x}\,dx = \int e^{u}\,\frac{du}{3} = \frac{1}{3}\int e^{u}\,du = \frac{1}{3}e^{u} + c = \frac{1}{3}e^{3x} + C$

Check: $\frac{d}{dx}\left(\frac{1}{3}e^{3x} + c\right) = \frac{1}{3}e^{3x}(3) = e^{3x}$

→ To integrate $\int e^{\text{something linear in } x}\,dx$, use substitution!

Example 2: The number of bacteria in a Petri dish doubles every hour. If $g(t)$ gives the rate of change of population, in thousands of bacteria per hour, and the dish started with 10,000 bacteria, find a function $Q(t)$ that measures the # of bacteria at time t.

$g(t) = 2^t$. We want $Q(t) = \int 2^t \, dt = \int e^{\ln(2^t)} \, dt$

$= \int e^{t \ln(2)} \, dt$

$u = t \ln(2)$

$du = \ln(2) \, dt \Rightarrow \frac{du}{\ln(2)} = dt$

$= \frac{1}{\ln(2)} \int e^u \, du = \frac{e^{t \ln(2)}}{\ln(2)} + C = \frac{2^t}{\ln(2)} + C$. But $Q(0) = 10$

So $\frac{2^0}{\ln(2)} + C = 10 \Rightarrow C = 10 - \frac{1}{\ln(2)}$ and $\boxed{Q(t) = \frac{2^t}{\ln(2)} + 10 - \frac{1}{\ln(2)}}$

Example 3: $\int x^2 e^{4x^3}\,dx$

$u = 4x^3 \Rightarrow \frac{du}{dx} = 12x^2 \Rightarrow \frac{du}{12x^2} = dx$

$\int x^2 e^{4x^3}\,dx = \int \cancel{x^2}\, e^u \cdot \frac{du}{12\cancel{x^2}} = \frac{1}{12}\int e^u\,du$

$= \frac{1}{12} e^u + C = \frac{1}{12} e^{4x^3} + C$

Integrals that end up looking like $\int \frac{1}{u}\,du$

Know $\int \frac{1}{u}\,du = \ln|u| + C$

<u>Example</u>: $\int \tan\theta\,d\theta = \int \frac{\sin\theta}{\cos\theta}\cdot d\theta$

Try 1: let $u = \cos\theta$.

$du = -\sin\theta\,d\theta \Rightarrow d\theta = \frac{-du}{\sin\theta}$

$\int \frac{\sin\theta}{\cos\theta}\,d\theta = -\int \frac{\cancel{\sin\theta}}{u}\,\frac{du}{\cancel{\sin\theta}}$

$= -\int \frac{du}{u} = -\ln|u| + C =$

$-\ln|\cos\theta| + C$

Try 2: let $u = \sin\theta$

$du = \cos\theta\,d\theta \Rightarrow \frac{du}{\cos\theta} = d\theta$

$\int \frac{\sin\theta}{\cos\theta}\,d\theta = \int \frac{u}{\cos\theta}\cdot\frac{du}{\cos\theta}$

$= \int \frac{u\,du}{\cos^2\theta}$

Try 2: let $u = \sin\theta$

$$du = \cos\theta \, d\theta \Rightarrow \frac{du}{\cos\theta} = d\theta$$

$$\int \frac{\sin\theta}{\cos\theta} d\theta = \int \frac{u}{\cos\theta} \cdot \frac{du}{\cos\theta}$$

$$= \int \frac{u \, du}{\cos^2\theta}$$

Recall: $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \sin^2\theta$

$$= \int \frac{u \, du}{1 - \sin^2\theta} = \int \frac{u \, du}{1 - u^2}$$

let $v = 1 - u^2 \Rightarrow dv = -2u \, du$

$\Rightarrow \frac{dv}{-2u} = du$

$$= \int \frac{\cancel{u}}{v} \cdot \frac{dv}{(-2\cancel{u})} = -\frac{1}{2} \int \frac{dv}{v} = -\frac{1}{2} \ln|v| + C$$

$$= -\frac{1}{2} \ln|1 - u^2| + C = -\frac{1}{2} \ln|1 - \sin^2\theta| + C = -\frac{1}{2} \ln|(\cos\theta)^2| + C$$

$$= -\frac{1}{2}(2) \ln|\cos\theta| + C = -\ln|\cos\theta| + C$$

Example: $\int \frac{z-5}{z+12}\,dz$

$u = z+12 \Rightarrow du = dz$

$\Rightarrow z = u-12$

So $z-5 = u-12-5 = u-17$

$$= \int \frac{u-17}{u}\,du$$

$$= \int \frac{u}{u} - \frac{17}{u}\,du$$

$$= \int 1\,du - \int \frac{17}{u}\,du$$

$$= u - 17\ln|u| + C$$

$$= z+12 - 17\ln|z+12| + C$$

A formula: $\int \ln(x)\,dx = x\ln(x) - x + C$

Is it true? Let's check!

$$\frac{d}{dx}\left(x\ln(x) - x + c\right)$$

$$= x \cdot \frac{1}{x} + \ln(x)(1) - 1 + 0$$

$$= 1 + \ln(x) - 1$$

$$= \ln(x)$$

Yes! The formula is true, because $\frac{d}{dx}\left(x\ln(x) - x + c\right) = \ln(x)$. We have an antiderivative!