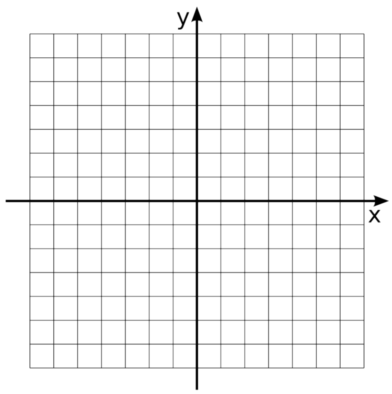
Final Review - Chapter 2
(Limits, + Continuity + L'Hospital's Rule)

Example 1: Sketch the graph of $f(x) = \begin{cases} \sqrt{-x}, & \text{if } x < 0 \\ x^2 \text{ if } 0 < x \leq 2 \\ x - 5, & \text{if } x > 2 \end{cases}$ and give the interval on which f is continuous. At what numbers is f continuous from the right, left or neither?



a) $\lim\limits_{x \to 0^-} f(x)$

b) $\lim\limits_{x \to 0^+} f(x)$

c) $\lim\limits_{x \to 0} f(x)$

d) $\lim\limits_{x \to 2^-} f(x)$

e) $\lim\limits_{x \to 2^+} f(x)$

f) $\lim\limits_{x \to 2} f(x)$

- Find limits using factoring, algebra, conjugates.

Example 2: Find the following limits:

a) $\lim\limits_{x \to -1^-} f(x)$ for $f(x) = \begin{cases} x^2 - 1 & \text{for } x < 1 \\ 2x + 3 & \text{for } x \geq 1 \end{cases}$

b) $\lim\limits_{x \to 0^+} f(x)$ where $f(x) = \begin{cases} x^2 + 4 & \text{for } x > 0 \\ 2\cos(x) + 5 & \text{for } x \leq 0 \end{cases}$

Example 3: Find the following limits:

a) $\lim\limits_{x \to 1} e^{x-1} \sin\left(\frac{\pi x}{2}\right)$

b) $\lim\limits_{x \to 0} \frac{5x^2}{1 - \cos x}$

Example 4: Find the following limits:

a) $\lim_{x \to 3} \frac{2x^2 - 18}{x^2 + x - 12}$

b) $\lim_{h \to 0} \frac{(4+h)^3 - 64}{h}$

Example 5: Find the following limits:

a) $\lim_{x \to -4} \frac{\frac{1}{4} + \frac{1}{x}}{4 + x}$

b) $\lim_{x \to -4} \frac{\sqrt{x^2 + 9} - 5}{x^2 + 2x - 8}$

- Find infinite limits. As in the limit is equal to plus or minus infinity or has an infinite discontinuity.
- Find limits at infinity. This means x goes to plus or minus infinity.

Example 6: Find the following limits:

a) $\lim_{x \to 5^-} \frac{e^x}{(x - 5)^3}$

b) $\lim_{x \to \pi^-} \cot x$

Example 7: Find the following limits.

a) $\displaystyle\lim_{x\to\infty} \frac{4x^4+5}{(x^2-2)(2x^2-1)}$

b) $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{9x^6-x}}{x^3+1}$

Example 8: Find the following limits.

a) $\displaystyle\lim_{x\to\infty} \frac{x+x^3+x^5}{1-x^2+x^4}$

b) $\displaystyle\lim_{x\to\infty} \frac{x^3-2x+3}{5-2x^2}$

Example 9: Find the following limits.

a) $\displaystyle\lim_{x\to\infty} \sec\left(\frac{x^2}{x^3-2}\right)$

b) $\displaystyle\lim_{x\to 0^+} \arctan(1/x)$

Example 10: Find the following limits using l'Hospital's rule. I won't tell you explicitly to do this on the exam. You will have to know when you can/ cannot apply this rule.

a) $\lim\limits_{x\to\infty} \dfrac{1-e^x}{1+2e^x}$

b) $\lim\limits_{h\to 0} \dfrac{\sin h}{h\cos h}$

- Know and apply the defintion of continuity.
- Determine where a function is discontinuous and why.
- Determine the value of a constant that makes a function continuous.

Definition of Continuity A function f is continuous at c if the following three conditions are met:

1. ______________________________

2. ______________________________

3. ______________________________

Example 11: Find all points of discontinuity of $h(x) = \dfrac{x-4}{x^2-x-12}$ and explain why the points are discontinuous and state if they are removable or non-removable.

Example 12: Find the numbers, if any, at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither?

$$f(x) = \begin{cases} x^2+1 & \text{if } x < 0 \\ e^x & \text{if } 0 \leq x \leq 2 \\ 6x-7 & \text{if } x > 2 \end{cases}$$

Example 13: Determine the value of b such that the function $f(x) = \begin{cases} x^2+bx & x \leq 1 \\ 3\cos(\pi x) & x > 1 \end{cases}$ is continuous on the entire real line.

Example 14: Determine the values of a and b that will make the function $f(x) = \begin{cases} x+1 & \text{if } 1 < x < 3 \\ x^2+ax+b & \text{if } |x-2| \geq 1 \end{cases}$ continuous on the entire real number line.

Suggested Probs: Ch. 2 Rvw, Pg 165 # 1-19 odds, 29 and Pg 311 # 15 - 19 odds, 33