Final Review - Chapter 5
(Antiderivatives and Applications of Anti-Differentiation)

Example 1: Find the most general antiderivative of the function.

a) $g(x) = \dfrac{1}{x} + \dfrac{1}{x^2+1}$ b) $f(x) = \dfrac{x^2+\sqrt{x}}{x}$

Example 2: Given $f''(x) = 5x^3 + 6x^2 + 2$, $f(0) = 3$, $f(1) = -2$, find $f(x)$.

Example 3: A particle is moving with $v(t) = 2t - 1/(1+t^2)$ and $s(0) = 1$. Fin the position of the particle.

Example 4: Estimate the area under the curve $y = x^2 + 2$ on the interval $[0, 8]$ using 4 sub-intervals and the method given below.

a) left endpoints.

b) midpoints.

Example 5: Evaluate the following definite integrals.

a) $\displaystyle\int_0^{\pi/4} \frac{\sec^2 t}{\tan t + 1} dt$

b) $\displaystyle\int_1^4 \frac{x-2}{\sqrt{x}} dx$

Example 6: Find the most general anti-derivaatives.

a) $\displaystyle\int \left(\sec x \tan x + \frac{2}{\sqrt{1-x^2}}\right) dx$

b) $\displaystyle\int \frac{x}{(x-2)^3} dx$

Example 7: Find the most general anti-derivatives.

a) $\displaystyle\int \frac{\sin(1/x)}{x^2} dx$

b) $\displaystyle\int \frac{\cos^{-1} x}{\sqrt{1-x^2}} dx$

Example 8: Find the derivative of the following functions.

a) $F(x) = \displaystyle\int_2^{x^3} \sqrt{1+t^4}\, dt$

b) $H(x) = \displaystyle\int_{e^x}^{x^2} \sec t\, dt$

Example 9: A particle moves along a line with velocity function $v(t) = 2\sin t$, where v is measured in meters per second.

(a) Find the displacement over the time interval $[0, 6]$

(b) Find the total distance traveled during the time interval $[0, 6]$

Example 10: A bacteria population is 4000 at time $t = 0$ and its rate of growth is 1000×2^t bacteria per hour after t hours. What is the population after one hour?