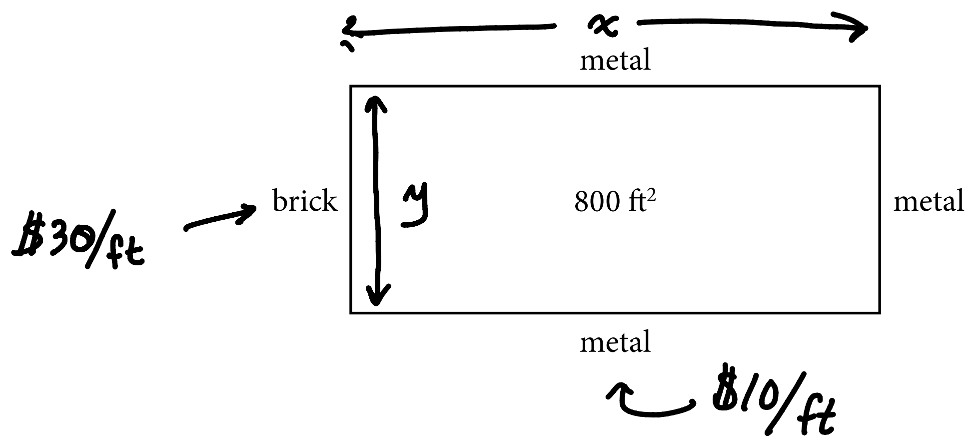
6. (10 points) A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing \$30 per foot and on the other three sides with a metal fence costing \$10 per foot. The area of the garden is to be 800ft^2. What are the dimensions of the garden that minimize the cost of the fencing? (For full credit, you must justify your answer.)



area $= A = xy = 800$
or
$y = 800x^{-1}$

goal: minimize cost

cost $= C = 30y + 10y + 2(10)(x) = 40y + 20x$.

So $C(x) = 40(800x^{-1}) + 20x = 32000x^{-1} + 20x$.

Now $C'(x) = -32000x^{-2} + 20x = 0$

$20x = \dfrac{32000}{x^2}$ or $x^3 = \dfrac{32000}{20} = 1600$

So $x = 40$.

First Der. Test: $-\,-\,-\;0\;+\,+\,+$ ← sign C'

0 1 40 10,000

$C(x)$ has a local min at $x = 40$.

Is $x = 40$ a global min?

option1: Yes. Because $x=40$ is the only crit. point in the domain in which $C(x)$ is continuous.

option2: Yes. Because $C''(x) = 64000x^{-3} + 20$ which is always positive ~~on~~ in the domain. So $C(x)$ is ccup.

answer the question

dimensions: $x = 40$ ft
$y = 20$ ft.

7. (12 points) Let $g(x) = \dfrac{e^x}{1+x}$. Note first and second derivatives are

$$g'(x) = \frac{xe^x}{(1+x)^2} \quad \text{and} \quad g''(x) = \frac{e^x(x^2+1)}{(1+x)^3}.$$ ← always +

(a) Evaluate the following limits.

i. $\lim\limits_{x\to\infty} g(x) = \lim\limits_{x\to\infty} \dfrac{e^x}{1+x} \overset{(H)}{=} \lim\limits_{x\to\infty} \dfrac{e^x}{1} = \infty$

ii. $\lim\limits_{x\to-\infty} g(x) = \lim\limits_{x\to\infty} \dfrac{e^{-x}}{1-x} = \lim\limits_{x\to\infty} \dfrac{1}{e^x(1-x)} = 0$ since $e^x(1-x) \to -\infty$.

iii. $\lim\limits_{x\to-1^-} g(x) = \lim\limits_{x\to-1^-} \dfrac{e^x}{1+x} = -\infty$ since $e^x \to \frac{1}{e}$ and $1-x \to 0^-$.

(b) Sketch the graph of $g(x)$. Label any asymptotes, x- and y-intercepts, local minimums and local maximums, and inflection points, if appropriate.

Note (ii) and (iii) imply $y=0$ is a horizontal asymptote and $x=-1$ is a vertical asymptote.

$g'(x) = 0$ when $x = 0$

$-$ ⋮ $-$ 0 $+$ ← sign g'

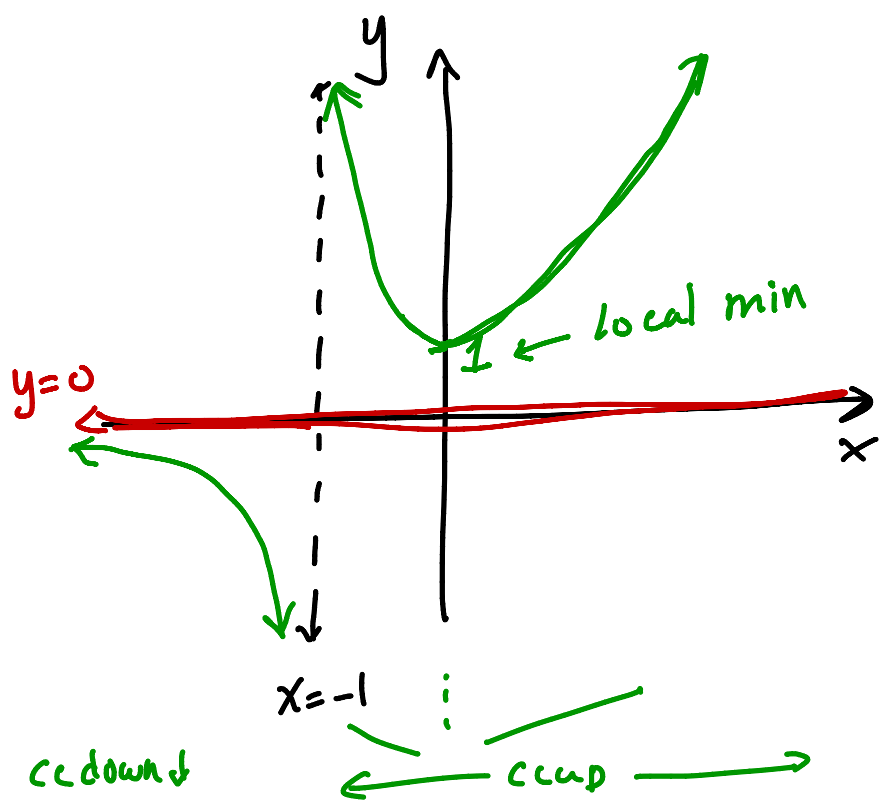
-1 $\quad$ 0

$g'' \neq 0$.

$- - -$ ⋮ $+ + +$ ← sign g''

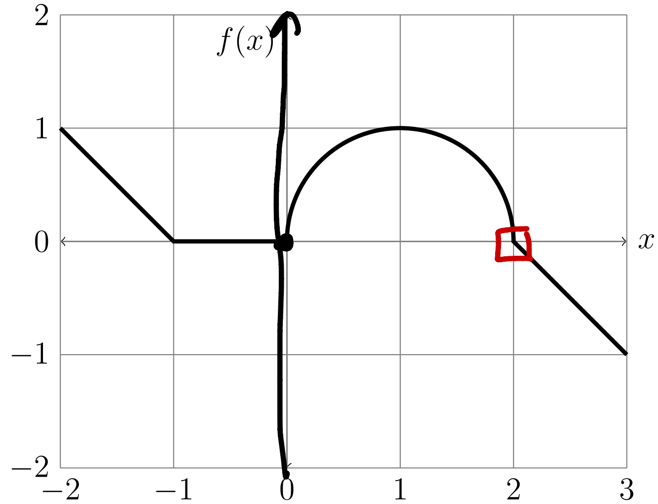
-1

cc down ⋮ cc up



9. (10 points) The function $f(x)$ has been graphed below. The curve for $0 < x < 2$ is an upper half circle. Define a new function $g(x)$, as

$$g(x) = \int_0^x f(s)\, ds.$$



Use the graph above to answer the questions below.
Note: Pay attention to whether question concerns the function f, f', g or g'.

(a) What is the value of $f(0)$?

$f(0) = 0$

(b) What is the value of $g(3)$?

signed area under curve from $x=0$ to $x=3$

Ans: $\frac{1}{2}\pi(1)^2 - \frac{1}{2}(1)(1)$
$= \frac{1}{2}\pi - \frac{1}{2} = \frac{1}{2}(\pi - 1)$

(c) What is the value of $g(-2)$?

$g(-2) = \int_0^{-2} f(s)ds = -\int_{-2}^{0} f(s)ds = -\frac{1}{2}$ area under curve.

(d) What is the value of $f'(2)$?

DNE.
A corner at $x=2$. So $f'(2)$ is undefined.

(e) What is the value of $g'(1)$?

$g'(x) = f(x)$ by FTC part I.

Ans: $g'(1) = f(1) = 1$