Solutions

Lecture Notes: §1.4

1. Use the Laws of Exponents to rewrite and simplify. Write down the rules that you are using to the side of your work.

(a) $(25^2)(5^{-3})$

$= \dfrac{25^2}{5^3} = \dfrac{25^2}{25 \cdot 5}$

$= \dfrac{25}{5} = 5$

use

- $a^{-p} = \dfrac{1}{a^p}$
- $\dfrac{a^p}{a^q} = a^{p-q}$

(b) $\sqrt[3]{x^{-2}}$ $= \sqrt[3]{\dfrac{1}{x^2}} = \left(\dfrac{1}{x^2}\right)^{1/3}$

$= \dfrac{1}{x^{2/3}} = x^{-2/3}$

use

- $\sqrt[n]{a} = a^{\frac{1}{n}}$
- $(a^p)^q = a^{pq}$

(c) $b^{(n-1)}(3b^2)^n$

$= b^{n-1} \cdot 3^n \cdot b^{2n}$

$= 3^n \cdot b^{2n+n-1}$

$= 3^n b^{3n-1}$

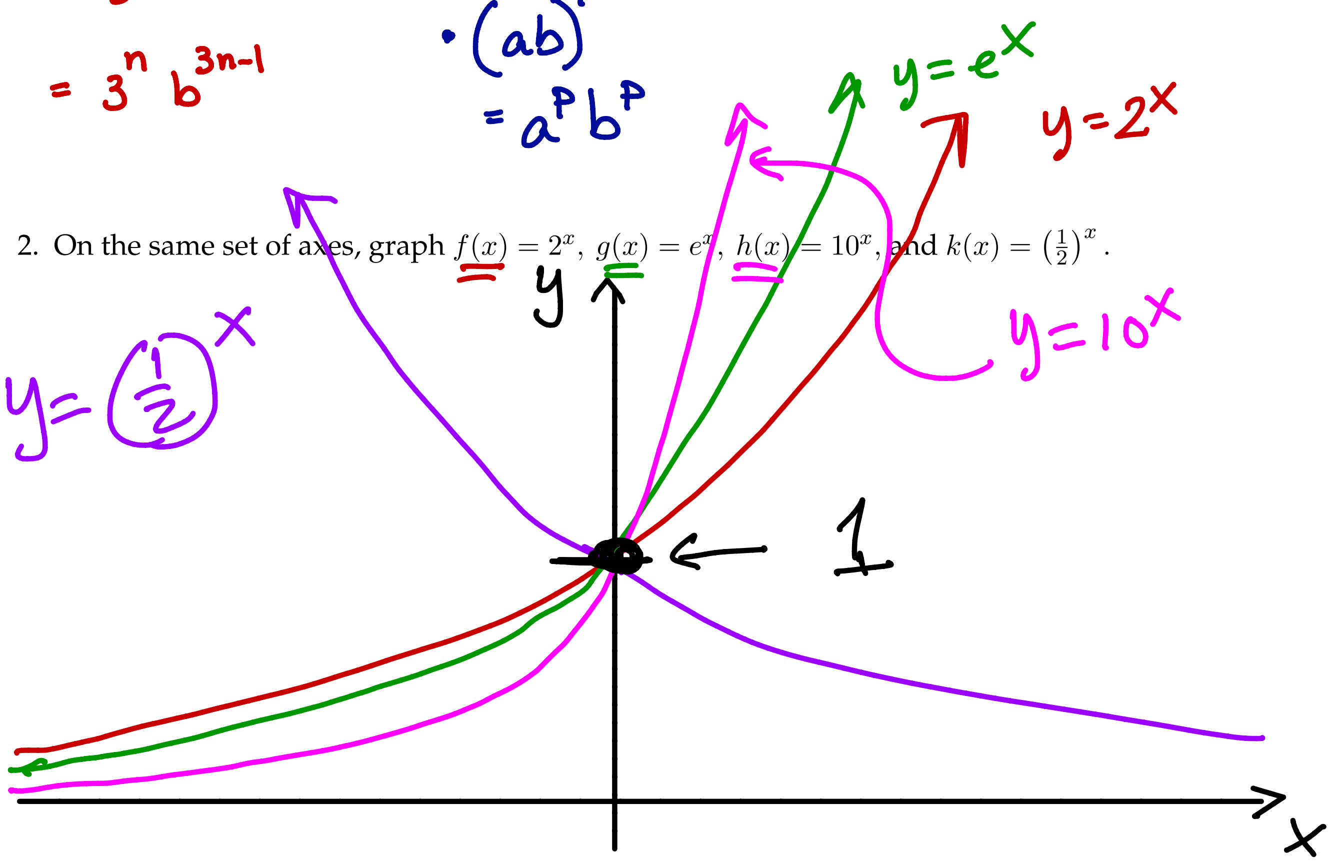
use

- $a^p a^q = a^{p+q}$
- $(ab)^p = a^p b^p$

d. $\dfrac{6x^2y}{\sqrt{4xy^3}}$ $= \dfrac{6x^2y}{2x^{1/2}y^{3/2}} = \dfrac{3x^{2-\frac{1}{2}}}{y^{\frac{3}{2}-1}} = \dfrac{3x^{3/2}}{y^{1/2}}$

$= 3x^{3/2}y^{-1/2}$

2. On the same set of axes, graph $f(x) = 2^x$, $g(x) = e^x$, $h(x) = 10^x$, and $k(x) = \left(\frac{1}{2}\right)^x$.



3. Assume $a > 0$. What is the domain and range of $f(x) = a^x$? Asymptotes?

domain: $(-\infty, \infty)$ asymptotes: $y=0$ or x-axis

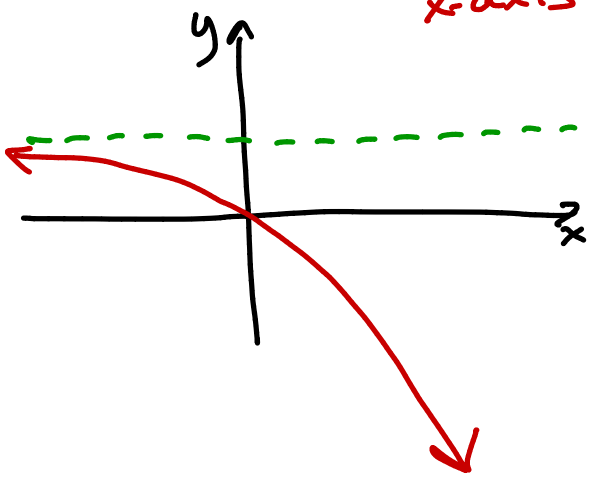
range: $(0, \infty)$

4. Graph each function below using what you know about transformations of functions. Determine its domain and range.

(a) $f(x) = 1 - 2^x$

shift up

reflect about x-axis



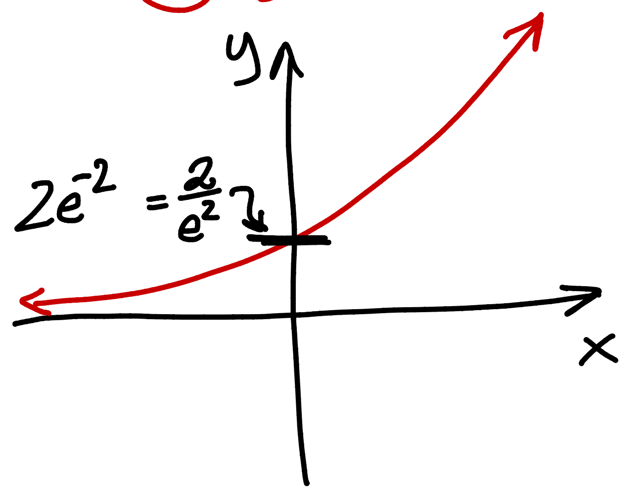
domain: $(-\infty, \infty)$

range: $(-\infty, 1)$

(b) $y = 2e^{x-2}$

shift right

stretch vertically



5. Are the following statements true or false? If either case, explain why. If possible, change the false statements so that they are a true statement.

F a. $(a+b)^2 = a^2 + b^2$ $\quad (a+b)^2 = a^2 + 2ab + b^2$

F b. $\sqrt{x^2+4} = x+2$ $\quad$ $\boxed{\text{Note } \sqrt{3^2+4^2} = \sqrt{25} = 5 \quad 3+4 = 7}$

F c. $\dfrac{a+b}{c+d} = \dfrac{a}{c} + \dfrac{b}{d}$ $\quad \dfrac{2+4}{1+1} = \dfrac{6}{2} = 3$ but $\dfrac{2}{1} + \dfrac{4}{1} = 6$

T d. $\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$ $\quad \dfrac{a+b}{c} = \dfrac{1}{c}(a+b) = \dfrac{1}{c} \cdot a + \dfrac{1}{c} \cdot b = \dfrac{a}{c} + \dfrac{b}{c}$