

SECTION 2-7: DERIVATIVES AND RATES OF CHANGE

1. Given the curve $y = g(x)$,

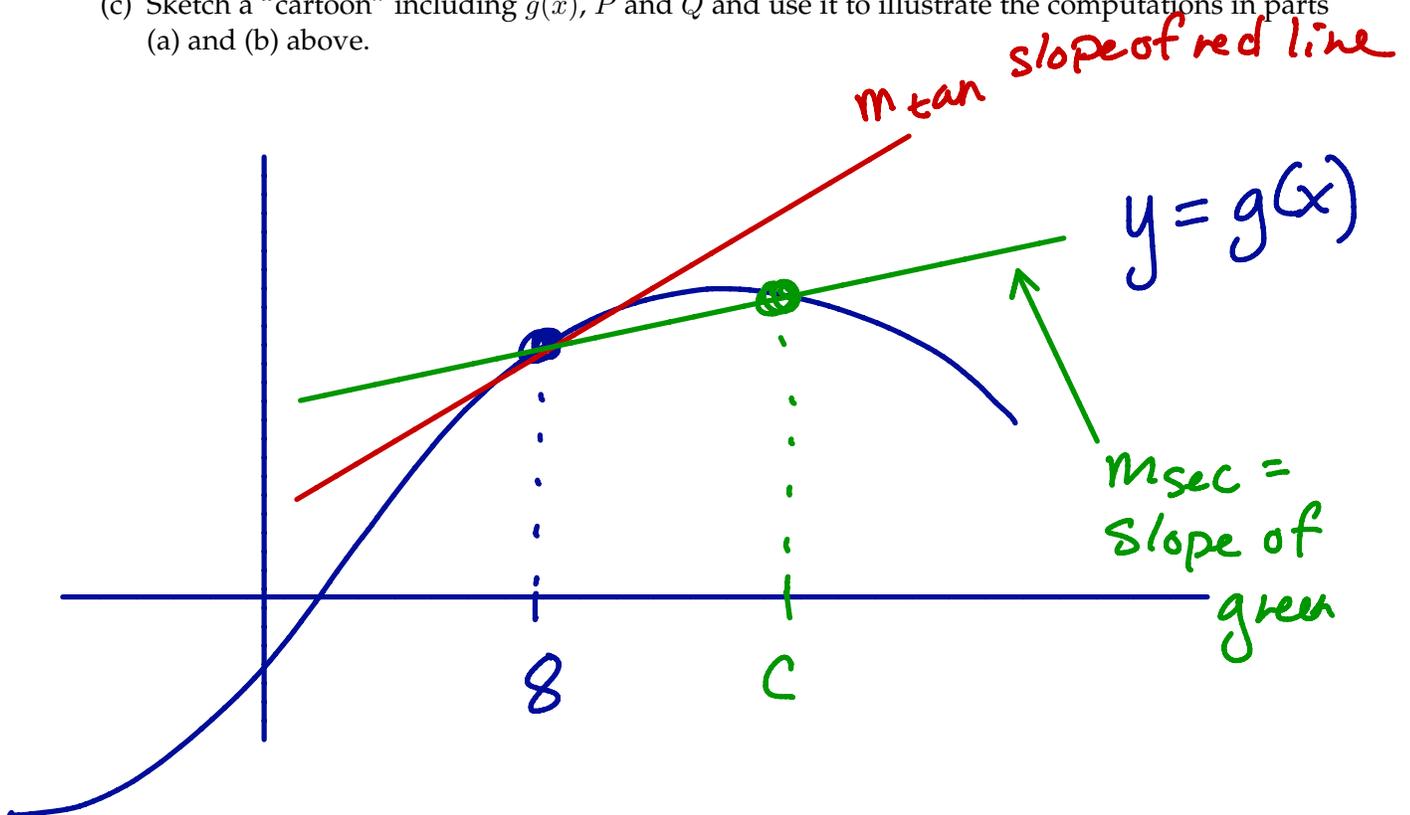
(a) Write an expression for the slope of the secant line through the points $P(8, g(8))$ and $Q(c, g(c))$.

$$m_{\text{sec}} = \frac{g(c) - g(8)}{c - 8} = \frac{g(8) - g(c)}{8 - c}$$

(b) Write an expression for the slope of the tangent line at $P(8, g(8))$.

$$m_{\text{tan}} = \lim_{c \rightarrow 8} \frac{g(c) - g(8)}{c - 8}$$

(c) Sketch a "cartoon" including $g(x)$, P and Q and use it to illustrate the computations in parts (a) and (b) above.



2. (a) **Fill in the boxes** The derivative of a function f at a number a is:

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

provided the limit exists!

(b) Use the expression above to find $f'(2)$ for $f(x) = 6x - 3x^2$.

$$\begin{aligned} f'(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{6(2+h) - 3(2+h)^2 - [12 - 12]}{h} \\ &= \lim_{h \rightarrow 0} \frac{12 + 6h - 3(4 + 4h + h^2)}{h} = \lim_{h \rightarrow 0} \frac{-6h - 3h^2}{h} = \lim_{h \rightarrow 0} -6 - 3h = -6 \end{aligned}$$

(c) Find $f(2)$. $f(2) = 6 \cdot 2 - 3 \cdot 2^2 = 12 - 12 = 0$

(d) Use the answers to parts (a) and (b) to write an equation of the line tangent to $f(x)$ when $x = 2$.

For equ. of line, I need point: $(2, 0)$

slope: -6 .

line: $y - 0 = -6(x - 2)$ or $y = -6(x - 2)$

(e) Sketch a "cartoon" including $f(x)$ and that tangent line. Is your answer in part (c) plausible?

