shorthand

1. Complete **The Product Rule:** If f and g are differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\cdot\left[\frac{d}{dx}g(x)\right]+\left[\frac{d}{dx}f(x)\right]\cdot g(x) = f\cdot g' + f'\cdot g$$

2. Complete **The Quotient Rule:** If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot\left[\frac{d}{dx}(f(x))\right]-f(x)\cdot\left[\frac{d}{dx}[g(x)]\right]}{[g(x)]^2} = \frac{g\cdot f' - f\cdot g'}{(g)^2}$$

3. Find the derivatives for each function below. *Do not use the Product Rule or the Quotient Rule if you don't have to!*

   (a) $f(x) = (1-x^2)(e^x + x)$

   $$f'(x) = (1-x^2)(e^x+1) + (-2x)(e^x+x)$$

   (b) $g(x) = \frac{\sqrt{x}}{8}(1-x\sqrt{x}) = \frac{1}{8}\left(x^{1/2} - x^2\right)$

   $$g'(x) = \frac{1}{8}\left(\frac{1}{2}x^{-1/2} - 2x\right)$$

   (c) $h(x) = \frac{10x - x^{3/2}}{4x^2} = \frac{5}{2}x^{-1} - \frac{1}{4}x^{-1/2}$

   $$h'(x) = -\frac{5}{2}x^{-2} + \frac{1}{8}x^{-3/2}$$

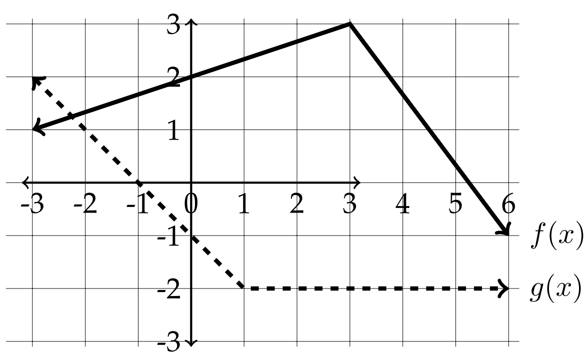
(d) $y = \dfrac{\sqrt[3]{x}}{2x+1} = \dfrac{x^{1/3}}{2x+1}$

$$y' = \frac{(2x+1)\left(\frac{1}{3}x^{-2/3}\right) - x^{1/3}(2)}{(2x+1)^2} \cdot \frac{3x^{2/3}}{3x^{2/3}} = \frac{(2x+1) - 6x}{3x^{2/3}(2x+1)^2} = \frac{1-4x}{3x^{2/3}(2x+1)^2}$$

(e) $v(t) = \dfrac{2te^t}{t^2+1}$

$$v'(t) = \frac{(t^2+1)\cdot \boxed{\frac{d}{dt}(2te^t)} - 2te^t(2t)}{(t^2+1)^2} = \frac{(t^2+1)\boxed{\left[2te^t + 2e^t\right]} - 4t^2e^t}{(t^2+1)^2}$$

4. The graphs of $f(x)$ (shown thick) and the graphs of $g(x)$ (shown dashed) are shown below. If $h(x) = f(x)g(x)$, find $h'(0)$.



$$h'(0) = f(0)\cdot g'(0) + f'(0)\cdot g(0)$$
$$= 2\cdot(-1) + \frac{1}{3}\cdot -1$$
$$= -7/3$$

5. Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$ and $g'(5) = 2$. Find the following values.

(a) $(f-g)'(5)$

$$f'(5) - g'(5)$$
$$= 6 - 2$$
$$= 4$$

(b) $(fg)'(5)$

$$f(5)\cdot g'(5) + g(5)\cdot f'(5)$$
$$= 1\cdot 2 + (-3)(6)$$
$$= 2 - 18 = -16$$

(c) $(g/f)'(5)$

$$= \frac{f(5)\cdot g'(5) - g(5)\cdot f'(5)}{f(5)^2}$$
$$= \frac{1\cdot 2 - (-3)(6)}{1^2}$$
$$= 2 + 18 = 20$$