1. Fill in the table below.

Derivatives of Trigonometric Functions:

- $\frac{d}{dx}(\sin x) = \underline{\cos x}$
- $\frac{d}{dx}(\cos x) = \underline{-\sin x}$
- $\frac{d}{dx}(\tan x) = \underline{\sec^2 x}$
- $\frac{d}{dx}(\csc x) = \underline{-\csc x \cot x}$
- $\frac{d}{dx}(\sec x) = \underline{\sec x \tan x}$
- $\frac{d}{dx}(\cot x) = \underline{-\csc^2 x}$

2. Derive the formula for $\frac{d}{dx}[\tan(x)]$.

$$\frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

using $\sin^2 x + \cos^2 x = 1$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

3. Find the derivative of $y = \dfrac{\sec x}{1 - x\tan x}$.

$$y' = \frac{(1 - x\tan x)(\sec x \tan x) - (\sec x)\left(0 - \frac{d}{dx}(x\tan x)\right)}{(1 - x\tan x)^2}$$

$$= \frac{(1 - x\tan x)(\sec x \tan x) + (\sec x)[1(\tan x) + (x)(\sec^2 x)]}{(1 - x\tan x)^2}$$

$$= \frac{(1 - x\tan x)(\sec x \tan x) + \sec x \tan x + x\sec^3 x}{(1 - x\tan x)^2}$$

4. If $f(\theta) = e^{\theta}\cos(\theta)$, find $f''(\theta)$.

$$f'(\theta) = e^{\theta}\cos\theta + e^{\theta}(-\sin\theta)$$
$$= e^{\theta}(\cos\theta - \sin\theta)$$

$$f''(\theta) = e^{\theta}(\cos\theta - \sin\theta) + e^{\theta}(-\sin\theta - \cos\theta)$$
$$= e^{\theta}(\cos\theta - \sin\theta - \sin\theta - \cos\theta)$$
$$= -2e^{\theta}\sin\theta$$

5. Find $\frac{d}{dt}[t \sin t \cos t]$.

$$\frac{d}{dt}[\underbrace{t\sin t}_{f}\,\underbrace{\cos t}_{g}] = \overset{f'}{\frac{d}{dt}[t\sin t]}\cdot\overset{g}{(\cos t)} + \overset{f}{(t\sin t)}\cdot\overset{g'}{(-\sin t)}$$

$$= [1\cdot\sin t + t(\cos t)](\cos t) - t\sin^2 t$$

$$= \sin t\cos t + t\cos^2 t - t\sin^2 t$$

6. An elastic band is hung on a hook and a mass is hung on the lower end of the band. When the mass is pulled down 2 cm past its rest position and then released, it vibrates vertically. The equation of motion is

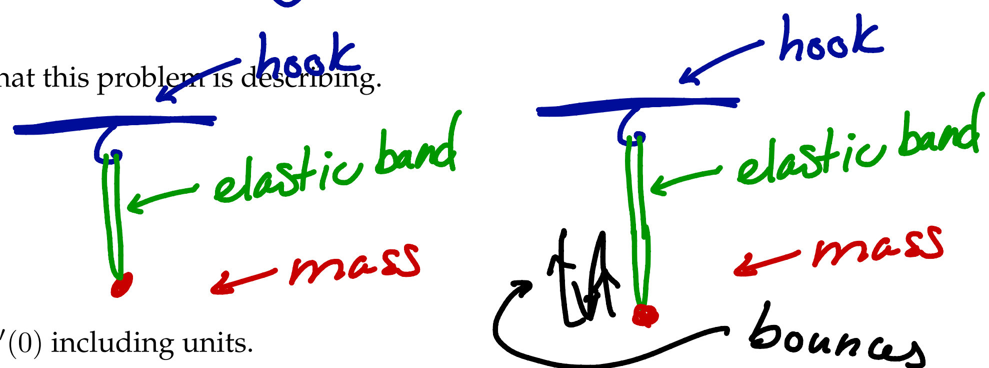
$$s = 2\cos t + 3\sin t, \quad \text{for } t \geq 0,$$

where s is measured in centimeters and t is measured in seconds. (We are taking the positive direction to be downward.)

(a) Why might you expect to use sines and cosines to model this particular problem?

Sines + cosines go back + forth (or up + down) just like a bouncing mass.

(b) Sketch a cartoon of what this problem is describing.



(c) Find $s(0)$, $s'(0)$, and $s''(0)$ including units.

$s(0) = 2$ cm

$s' = -2\sin t + 3\cos t$

$s'(0) = 3$ cm/s

$s'' = -2\cos t - 3\sin t$

$s''(0) = -2$ cm/s^2

(d) What does $s(0)$ tell you about the mass in the context of the problem?

The mass starts 2 cm beyond zero or resting... just like the problem says!

(e) What does $s'(0)$ tell you about the mass in the context of the problem?

When time starts, the velocity of the mass is 3 cm/s, So the mass is released with initial velocity in the downward direction.

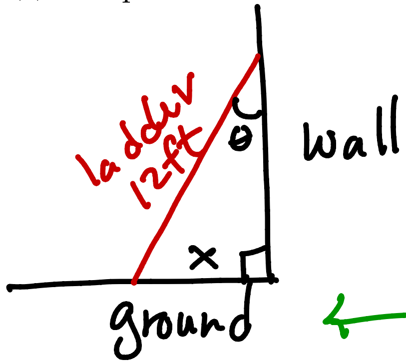
(f) What does $s''(0)$ tell you about the mass in the context of the problem?

When time starts, the acceleration of the mass is -2 cm/s^2. So the band is causing the velocity to decrease or you can think of the band as pulling up on the mass.

↙ Draw a picture.

7. A 12 foot ladder rests against a wall. Let θ be the angle between the ladder and the wall and let x be the distance from the base of the ladder and the wall.

   (a) Compute x as a function of θ.



$\sin\theta = \frac{x}{12}$ or

$x = 12\sin\theta$

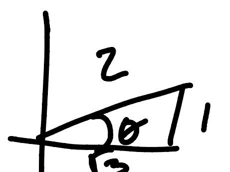
← when $\theta = \frac{\pi}{6}$, $x = 6$ (so ladder is 6 feet away)

   (b) How fast does x change with respect to θ when $\theta = \pi/6$? (Get an exact answer and a decimal approximation.)

$\frac{dx}{dt} = 12\cos\theta$, $\left.\frac{dx}{dt}\right|_{\theta=\frac{\pi}{6}} = 12\cos\left(\frac{\pi}{6}\right) = 12 \cdot \frac{\sqrt{3}}{2}$

$= 6\sqrt{3}$

≈ 10.39



$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

   (c) Interpret your answer from part (b) in the context of the problem. (Units will help you here.)

10.39 ft/radians

The rate of change of distance of the bottom of the ladder and the wall is increasing at a rate of 10.39 ft/rad when $\theta = \pi/6$

   (d) If the angle θ was *decreased* from $\pi/6$ radians to $\frac{\pi}{6} - \frac{1}{100}$ radians, estimate how the distance to the wall would change. Try to answer this question using only your answer from part b.

10.39 ft/radians means 0.1039 ft per $\frac{1}{100}$ radian

So we expect the distance to the wall to decrease by about 0.1 feet.