

SECTION 3.6: LOGARITHMIC DIFFERENTIATION

1. Find the derivative of

(a) $y = (3x - x^5)^{2/3}(x - \tan(x))^5$.

(b) Find the derivative of $y = (\sin(x))^x$.

SECTION 3.7: RATES OF CHANGE IN THE NATURAL AND SOCIAL SCIENCES

2. A ball is tossed straight up into the air. It has a velocity at time $t = 0$ seconds of 5 meters per second. It undergoes a constant acceleration due to gravity of -9.8 meters per second per second, m/s^2 . The height of the ball can be written in the form

$$h(t) = at + bt^2$$

where h is measured in meters, time is measured in seconds, and a and b are certain constants.

- (a) Determine the values for the constants.

- (b) What is the height of the ball at time $t = 0$? At $t = 1$?

- (c) At what times is the ball at height 0?

- (d) What is the average velocity of the ball over the time interval $[0.2, 0.21]$?

- (e) What is the average velocity of the ball over the time interval $[0.2, 0.201]$?

- (f) What is the instantaneous velocity of the ball at time $t = 0.2$??

- (g) At what time t is the ball motionless?

- (h) What is the velocity of the ball at time $t = 0$? At $t = 0.1$? At $t = 1$?

3. A stone is thrown in a pond and a circular ripple travels outward at a speed of 60 cm/s. Determine the rate of change of area inside the ripple at time $t = 1$ second and at time $t = 2$ seconds.

4. A population of bacteria starts at 500 cells and doubles every 30 minutes. Find a function $P(t)$ that describes this situation. Then compute the rate of change of the bacteria population at time $t = 60$ minutes.

5. A population of caribou is growing, and its population is

$$P(t) = 4000 \frac{3e^{t/5}}{1 + 2e^{t/5}}.$$

(a) What is the population at time $t = 0$?

(b) Determine the rate of change of the population at any time t .

(c) Determine the rate of change of the population at time $t = 0$ years.

(d) Determine the long term population.