1. Consider the function $f(x) = x^2$ on the interval $[-1, 3]$

   (a) Find the slope of the secant line of the graph of $f(x)$ from $x = -1$ to $x = 3$.

$$m_{sec} = \frac{f(3)-f(-1)}{3-(-1)} = \frac{9-1}{3+1} = \frac{8}{4} = \boxed{2}$$

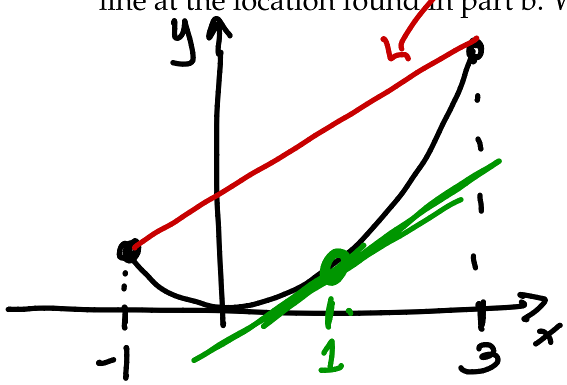
   (b) Find a value of x in $[-1, 3]$ where $f'(x)$ equals the value in part a.

$$f'(x) = 2x = \boxed{2}$$

So $x = 1$

Secant line $m=2$

   (c) Make a sketch of the graph of $f(x)$ and add to it the secant line from part a and the tangent line at the location found in part b. What property do the secant line and tangent line have?



$y = f(x) = x^2$

at $x=1$, $f'(x) = 2$

The green & red lines are parallel

$g(x) = x^{-1}$

2. Repeat Problem 1 with the function $g(x) = 1/x$ on $[1, 5]$.

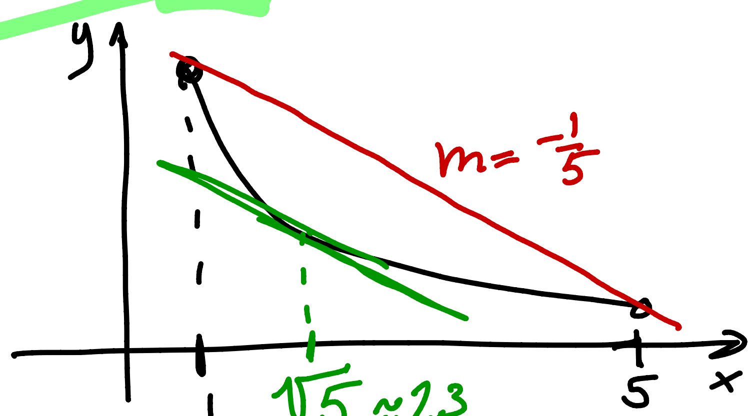
$$m_{sec} = \frac{g(5)-g(1)}{5-1} = \frac{\frac{1}{5}-1}{4} = \frac{-4}{5}\cdot\frac{1}{4} = \boxed{\frac{-1}{5}} = -0.20$$

$$g'(x) = -1x^{-2} = \boxed{\frac{-1}{5}}$$

So $\frac{-1}{x^2} = \frac{-1}{5}$

So $x = \pm\sqrt{5}$.

Only $x = \sqrt{5}$ in domain.

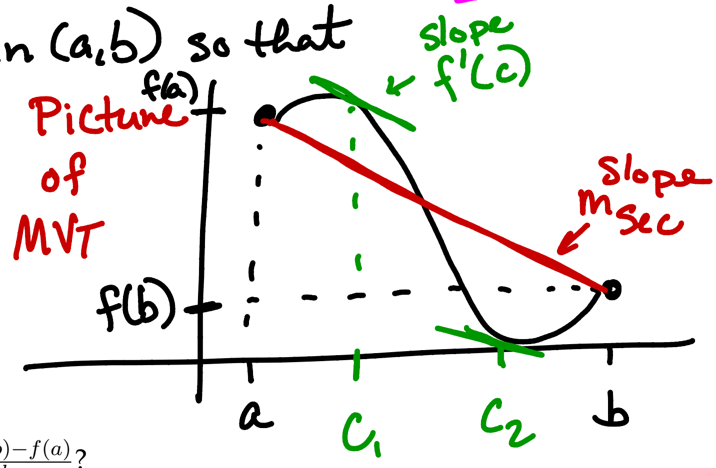


3. **Mean Value Theorem** $\boxed{\text{IF}}$ • f conts on [a,b]
• f differentiable on (a,b)

the graph of f is smooth + one piece.

$\boxed{\text{THEN}}$ there is an x-value c in (a,b) so that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

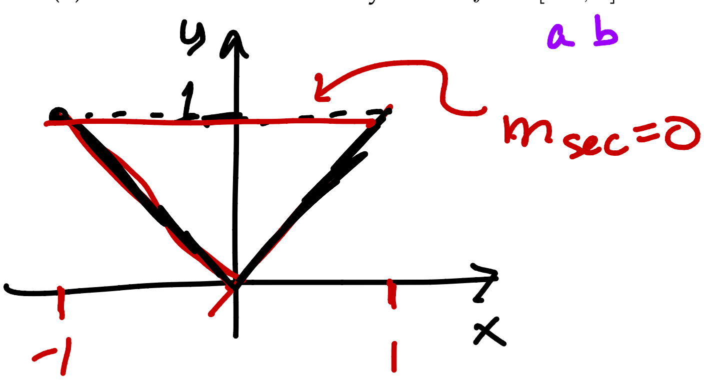


4. What is the *geometric* meaning of the value $\frac{f(b)-f(a)}{b-a}$?

It's the slope of secant.

5. Consider the function $f(x) = |x|$ on $[-1, 1]$.

(a) What would MVT say about f on $[-1, 1]$?



$$\frac{f(b)-f(a)}{b-a} = \frac{f(-1)-f(1)}{-1-1} = 0$$

MVT says there should be a c in (-1,1) where $f'(c)=0$.

(b) Does MVT "work" in this case? Why or why not?

There is no c so that $f'(c)=0$.

MVT does not apply! f(x) is NOT differentiable!

6. Suppose f is a continuous function on $[a, b]$ and $f'(x) > 0$ for every x in (a, b). How do $f(a)$ and $f(b)$ compare?

MVThm:

$$\frac{f(b)-f(a)}{b-a} = f'(c) > 0$$

Now $b-a>0$ always.

So $f(b)-f(a)>0$

So $f(b)>f(a)$

(The graph increases!)

7. Suppose f is a continuous function on $[a, b]$ and $f'(x) < 0$ for every x in (a, b). How do $f(a)$ and $f(b)$ compare?

$f(a) > f(b)$

8. Compare carefully the following two questions, then answer them.

(a) Suppose $f(x) = C$ on $[a, b]$, where C is a fixed constant. What can you say about $f'(x)$?

$f'(x) = 0.$ ← Just a derivative rule.

(b) Suppose $f(x)$ is continuous on $[a, b]$ and $f'(x) = 0$ on (a, b). What can you say about $f(x)$?

Part (a) tells us it could be a constant function but is it always?

$$\boxed{\frac{f(b)-f(a)}{b-a} = f'(c) = 0 \qquad \text{So } f(b)-f(a)=0 \text{ or } f(b)=f(a)}$$

But this is true for all x in $[a,b]$.

That is:

$$\frac{f(b)-f(x)}{b-x} = 0 \quad \text{so} \quad f(x)=f(b)$$

So $f(x) = f(b)$ ← some fixed constant.

9. Suppose a car is traveling down the road and in 30 minutes it travels 32.7 miles. What does the Mean Value Theorem have to say about this?

average velocity is $\frac{32.7 \text{ mi}}{30 \text{ min}} = \frac{32.7 \text{ mi}}{\frac{1}{2} \text{ hr}} = 65.4 \text{ mi/hr.}$

the equiv. of m_{sec}

MVT $\Rightarrow \exists$ some time in that 30 minutes in which the car's INSTANTANEOUS velocity is 65.4 mi/hr.

10. Suppose that $f(0) = -3$ and that $f'(x)$ exists and is less than or equal to 5 for all values of x. How large can $f(2)$ possibly be?

Know $f(0) = -3$ and $f'(x) \le 5$.

$$\frac{f(2) - f(0)}{2 - 0} = f'(c) \le 5$$

So $\frac{f(2) - (-3)}{2} \le 5$

So $f(2) + 3 \le 10$

So $f(2) \le 7$.

11. **Corollary 7:** If $f'(x) = g'(x)$ for all x in the interval (a, b), then

$$\boxed{f(x) = g(x) + C, \quad C \text{ fixed constant}}$$

Why? MVT to $H(x) = f(x) - g(x)$.

$H'(x) = f'(x) - g'(x) = 0$

8b tells us $H(x) = f(x) - g(x) = C$ or $f(x) = g(x) + C$