1. Follow the guidelines from the previous worksheet to sketch the graph of

$$f(x) = \frac{2}{x} + \ln(x).$$

a. What is the function's domain?

$x > 0$

b. Does this function have any symmetry?

none

c. Find a few choice values of x to evaluate the function at.

$f(1) = 2$

d. What behaviour occurs for this function at $\pm\infty$?

$$\lim_{x\to\infty} \frac{2}{x} + \ln(x) = 0 + \infty = \infty$$

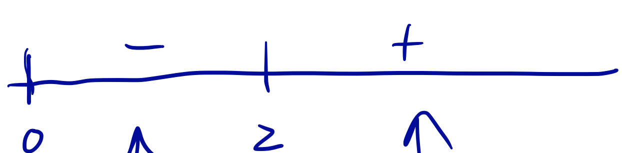
e. Does the function have any vertical asymptotes? Where?

$$\lim_{x\to 0^+} x\ln(x) = \lim_{x\to 0^+} \frac{\ln(x)}{1/x} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-1/x^2} = \lim_{x\to 0^+} -x = 0.$$

$$\lim_{x\to 0^+} \frac{2}{x} + \ln(x) = \lim_{x\to 0^+} \frac{1}{x}\left[2 + x\ln(x)\right] = \infty\,[2+0] = \infty$$

f. Find intervals where f is increasing/decreasing and identify critical points.

$$f'(x) = \frac{-2}{x^2} + \frac{1}{x} = \frac{x-2}{x^2}$$



decreasing

crit pt, local min

increasing

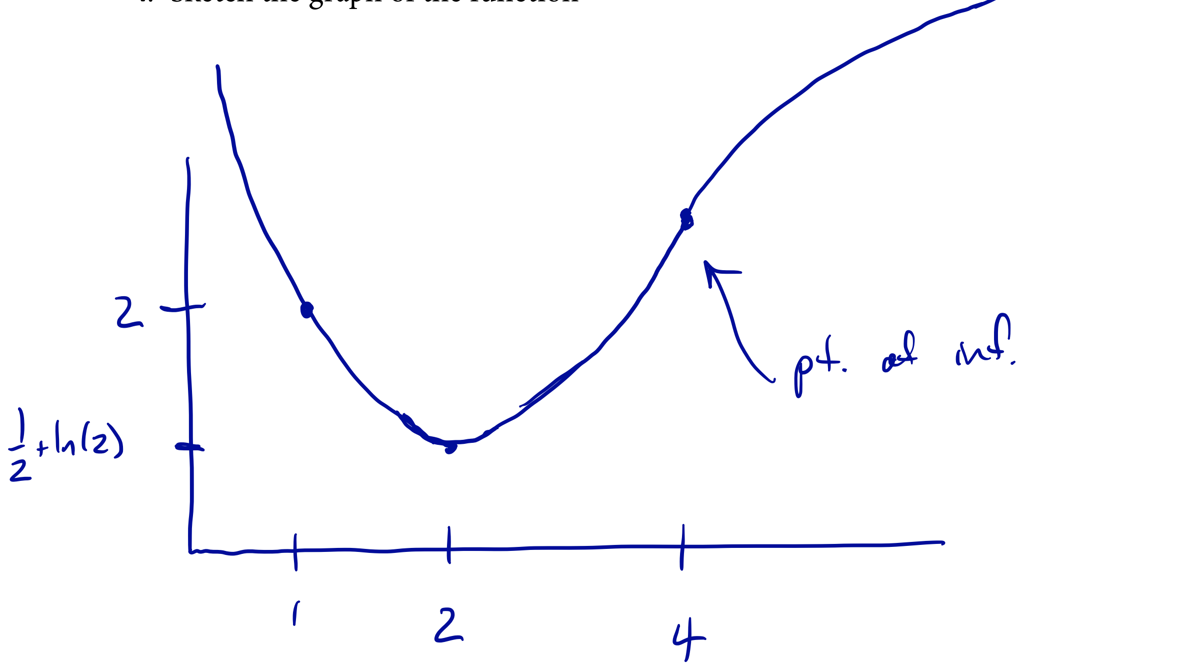
g. Classify each critical point as a local min/max/neither.

$x=2$ is the location of a local min

h. Find intervals where f is concave up/concave down and identify points of inflection

$$f''(x) = \frac{x^2 - 2x(x-2)}{x^4} = \frac{x - 2(x-2)}{x^3} = \frac{4-x}{x^3}$$

i. Sketch the graph of the function



2. Follow the guidelines from the previous worksheet to sketch the graph of

$$f(x) = x\sqrt{4 - x^2}.$$

a. What is the function's domain?

$-2 \leq x \leq 2$

b. Does this function have any symmetry?

odd symmetry

c. Find a few choice values of x to evaluate the function at.

$f(0) = 0, \quad f(\pm 2) = 0$

d. What behaviour occurs for this function at $\pm\infty$?

not defined near $\pm\infty$

e. Does the function have any vertical asymptotes? Where?

none

f. Find intervals where f is increasing/decreasing and identify critical points.

$$f'(x) = \sqrt{4-x^2} + \frac{x(-2x)}{2\sqrt{4-x^2}}$$

$$= \frac{4-x^2-x^2}{\sqrt{4-x^2}} = \frac{2(2-x^2)}{\sqrt{4-x^2}}$$

controls sign.

≥ 0

f': $-$ on $(-2, -\sqrt{2})$, $+$ on $(-\sqrt{2}, \sqrt{2})$, $-$ on $(\sqrt{2}, 2)$

$-2 \qquad -\sqrt{2} \qquad \sqrt{2} \qquad 2$

$-\sqrt{2}$: local min

$\sqrt{2}$: local max

g. Classify each critical point as a local min/max/neither.

see previous q.

h. Find intervals where f is concave up/concave down and identify points of inflection

$$f'(x) = \frac{2(2-x^2)}{\sqrt{4-x^2}}, \quad f''(x) = 2\left[\frac{-2x\sqrt{4-x^2} - (2-x^2)\frac{-x}{\sqrt{4-x^2}}}{(4-x^2)}\right]$$

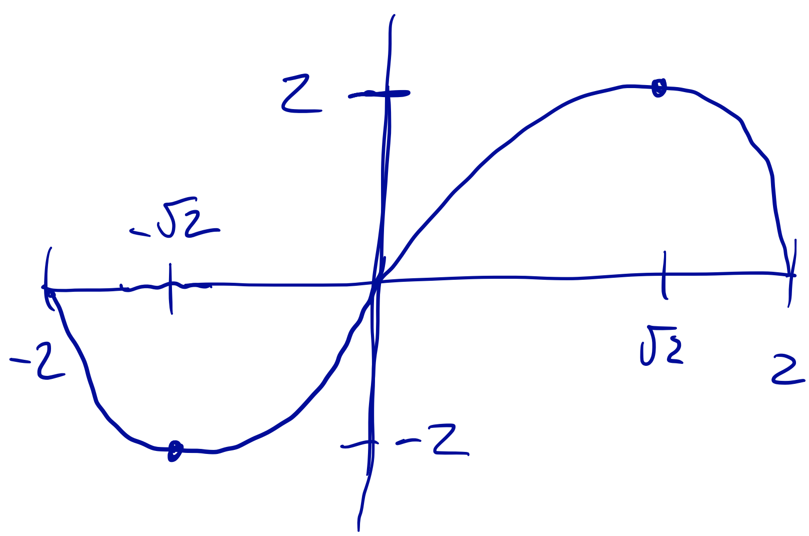
$$= 2\left[\frac{-2x + x[2-x^2]}{(4-x^2)^{3/2}}\right]$$

$$= 2\left[\frac{-x^3}{(4-x^2)^{3/2}}\right]$$

f'': -2 (+) (−) 2

c.u. c.d.

i. Sketch the graph of the function



3. Follow the guidelines from the previous worksheet to sketch the graph of

$$f(x) = \sin^{\cancel{3}\ 2}(x).$$

a. What is the function's domain?

$\mathbb{R}$

b. Does this function have any symmetry?

periodic, period is no more than 2π

c. Find a few choice values of x to evaluate the function at.

$f(x) = 0$ if $x = k\pi$ $k \in \mathbb{Z}$

$f(x) = 1$ if $x = \frac{\pi}{2} + k\pi$ $\pi \in \mathbb{Z}$

d. What behaviour occurs for this function at $\pm\infty$?

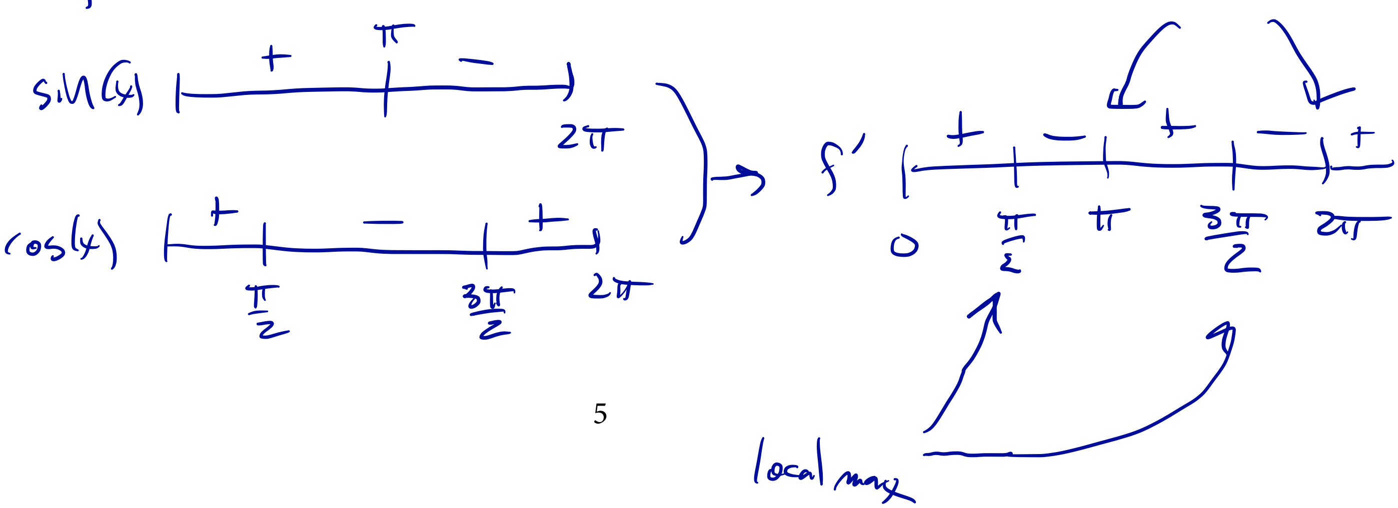
$\lim_{x \to \pm\infty} f(x)$ d.n.e.

e. Does the function have any vertical asymptotes? Where?

none

f. Find intervals where f is increasing/decreasing and identify critical points.

$f'(x) = 2\sin(x)\cos(x)$

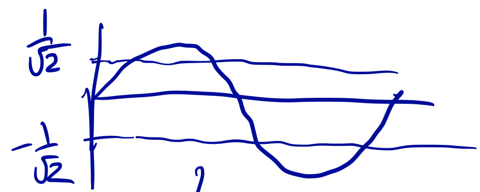


g. Classify each critical point as a local min/max/neither.

see prior

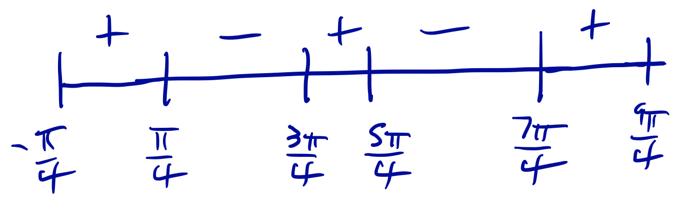
h. Find intervals where f is concave up/concave down and identify points of inflection

$$f''(x) = 2\left[\cos^2(x) - \sin^2(x)\right]$$
$$= 2\left[1 - 2\sin^2(x)\right]$$

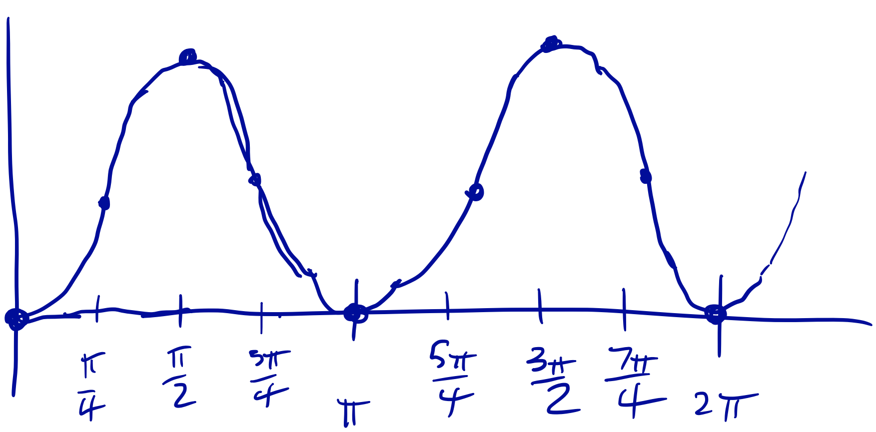


$f''(x) = 0$ when $\sin(x) = \pm\frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$



i. Sketch the graph of the function



[in fact, $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$!]

4. Follow the guidelines from the previous worksheet to sketch the graph of

$$f(x) = \frac{x}{\sqrt{9 + x^2}}.$$

a. What is the function's domain?

$\mathbb{R}$

b. Does this function have any symmetry?

odd

c. Find a few choice values of x to evaluate the function at.

$f(0) = 0$

d. What behaviour occurs for this function at $\pm\infty$?

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x}{x\sqrt{9/x + 1}} = \lim_{x\to\infty} \frac{1}{\sqrt{9/x+1}} = \frac{1}{\sqrt{0+1}} = 1$$

By odd symmetry: $\lim_{x\to-\infty} f(x) = -1.$

e. Does the function have any vertical asymptotes? Where?

None

f. Find intervals where f is increasing/decreasing and identify critical points.

$$f'(x) = \frac{1}{\sqrt{9+x^2}} - \frac{x \cdot (2x)}{2(9+x^2)^{3/2}}$$

$$= \frac{9}{(9+x^2)^{3/2}} > 0$$

always increasing

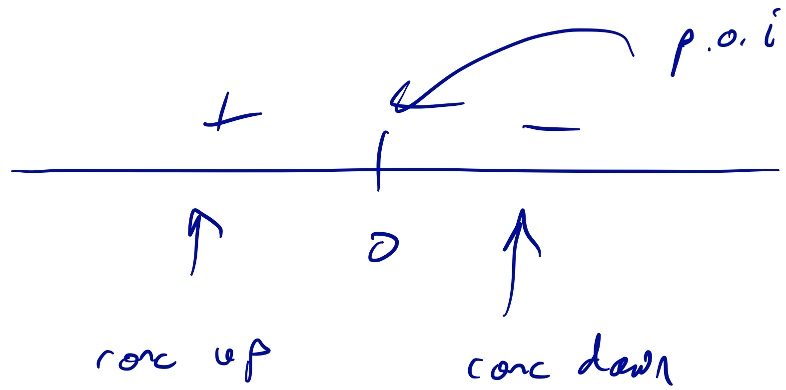
g. Classify each critical point as a local min/max/neither.

None

h. Find intervals where f is concave up/concave down and identify points of inflection

$$f''(x) = 9\left(\frac{-3}{2}\right)(9+x^2)^{-5/2}\cdot(2x)$$

$$= -27(9+x^2)^{-5/2}x$$



i. Sketch the graph of the function