5-4: Net Change

SECTION ~~5-3~~: ~~THE FUNDAMENTAL THEOREM OF CALCULUS~~

1. Compute $\int x^2(3-x)\,dx$

$$= \int (3x^2 - x^3)\,dx = \boxed{x^3 - \frac{1}{4}x^4 + C}$$

2. Compute $\int 9\sqrt{x} - 3\sec(x)\tan(x)\,dx$ $= \int (9x^{\frac{1}{2}} - 3\sec x \tan x)\,dx$

$$= 9 \cdot \frac{2}{3} x^{\frac{3}{2}} - 3\sec x + C$$

$$= 6x^{3/2} - 3\sec x + C$$

3. Find an antiderivative of $f(x) = \frac{1}{x^2}$ that does not have the form $-1/x + C$.

$$F(x) = \begin{cases} -\frac{1}{x} + 10 & \text{for } x > 0 \\ -\frac{1}{x} + \pi & \text{for } x < 0 \end{cases}$$

4. Snow is falling on my garden at a rate of

$$A(t) = 10e^{-2t}$$

kilograms per hour for $0 \leq t \leq 2$, where t is measured in hours.

(8) Find and interpret $A(1)$.

$A(1) = 10e^{-2} \approx 1.35$ kg/hr

(a) If $m(t)$ is the total mass of snow on my garden, how are $m(t)$ and $A(t)$ related to each other?

$m'(t) = A(t)$

(b) What does $m(2) - m(0)$ represent?

The mass of snow that fell during the 2 hour period ($t=0$, to $t=2$)

(c) Find an antiderivative of $A(t)$.

$-5e^{-2t}$

(d) Compute the total amount of snow accumulation from $t = 0$ to $t = 1$.

$\int_0^1 10e^{-2t}\,dt = -5e^{-2t}\Big|_0^1 = -5e^{-2} - (-5) = 5(1-e^{-2})$
$= 4.32$ kg

(e) Compute the total amount of snow accumulation from $t = 0$ to $t = 2$.

$\int_0^2 10e^{-2t}\,dt = -5e^{-2t}\Big|_0^2 = -5e^{-4} + 5 \approx 4.91$ kg

(f) From the information given so far, can you compute $m(2)$?

No. We don't know how much snow there was before $t=0$.

(g) Suppose $m(0) = 9$. Compute $m(1)$ and $m(2)$.

$m(1) = 9 + 4.32 = 13.32$ kg

$m(2) = 9 + 4.91 = 13.91$ kg

5. A airplane is descending. Its rate of change of height is $r(t) = -4t + \frac{t^2}{10}$ meters per second.

(a) if $A(t)$ is the altitude of the airplane in meters, how are $A(t)$ and $r(t)$ related?

$A'(t) = r(t)$

(b) What physical quantity does $\int_1^3 r(t)\,dt$ represent?

How much the plane's height changed in the 2 second intervat from $t=1$ to $t=3$.

(c) Compute $A(3) - A(1)$. $A(3) - A(1) = \int_1^3 r(t)dt = \int_1^3 \left(-4t + \frac{1}{10}t^2\right)dt$

$= -2t^2 + \frac{1}{30}t^3 \Big|_1^3 = \left(-2\cdot 3^2 + \frac{1}{30}3^3\right) - \left(-2 + \frac{1}{30}\right) = -15.13\ m$

(d) What is the height of plane when $t = 3$?
We don't know.

6. Gravel is being added to a pile at a rate of rate of $1 + t^2$ tons per minute for $0 \le t \le 10$ minutes. If $G(t)$ is the amount of gravel (in tons) in the pile at time t, compute $G(10) - G(0)$.

$\int_0^{10} (1+t^2)dt = t + \frac{1}{3}t^3 \Big|_0^{10} = 10 + \frac{1}{3}(1000) = 343$ tons