

RECITATION: REVIEW OF CHAPTERS 3 & 4

1. $f(x) = (x - 4)\sqrt[3]{x} = x^{4/3} - 4x^{1/3}$; $f'(x) = \frac{4(x-1)}{3x^{2/3}}$; $f''(x) = \frac{4(x+2)}{9x^{5/3}}$.

(a) Find the critical numbers of $f(x)$.

(b) Find the open intervals on which the function is increasing or decreasing.

(c) Classify all critical points – using the **first derivative test**.

(d) Classify all critical points – using the **second derivative test**.

From the other side: $f(x) = (x - 4)\sqrt[3]{x} = x^{4/3} - 4x^{1/3}$; $f'(x) = \frac{4(x-1)}{3x^{2/3}}$; $f''(x) = \frac{4(x+2)}{9x^{5/3}}$.

(e) Find the open intervals on which the function is concave up or concave down.

(f) Find the inflection points.

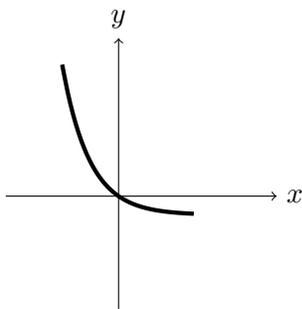
(g) Sketch the graph.

2. A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of $2 \text{ cm}^3/\text{sec}$, how fast is the water level rising when the water is 5 cm deep? (Volume of a cone is: $V = (1/3)\pi r^2 h$.)

3. Find the dimensions of the rectangle of maximum area that can be inscribed in an equilateral triangle of side 20 cm if one side of the rectangle lies on the base of the triangle.

4. Determine the absolute maximum and absolute minimum of $f(x) = \frac{1}{x} + x$ on $[1/2, 4]$.

5. Below is the graph of the DERIVATIVE, $h'(x)$ of $h(x)$.



(a) Does $h(x)$ have any critical points? If so, what are they?

(b) Does $h(x)$ have any local extrema? If so, where do they occur and what type (min/max)?

(c) Can you determine if $h(x)$ is concave up or down?

6. A circular metal plate is being heated in an oven. The radius of the plate is increasing at a rate of 0.01 cm/min when the radius is 50cm. How fast is the area of the plate increasing?

7. A Norman window is has a rectangular base and a semi-circle on top. What dimensions of the window minimize the perimeter if the area of the window is to be 4 ft^2 .

8. Find the domain of the function $f(x) = \frac{\sin(5x)}{x^2+x}$ and identify any vertical or horizontal asymptotes. Justify your answers.