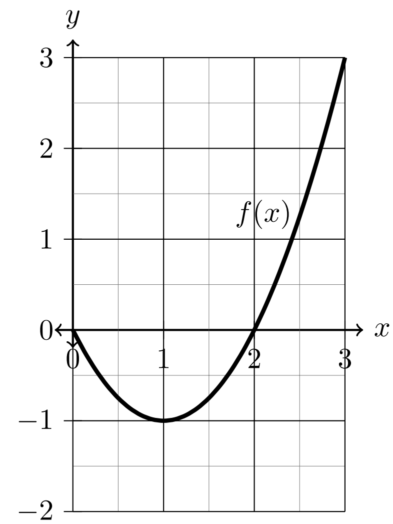
Definite Integrals and Areas "under" Curves

1. Estimate the area under $f(x) = x^2 - 2x$ on $[1, 3]$ with $n = 4$ using the
   (a) Right-hand endpoints



   (b) Left-hand endpoints

> **Definition of a Definite Integral** If f is a function defined for $a \leq x \leq b$, we divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = (b-a)/n$. We let $x_0 = (a), x_1, x_2, \cdots, x_n = (b)$ be the endpoints of these subintervals and we let $x_1^*, x_2^*, \cdots, x_n^*$ be **sample points**[1] in these subintervals, so x_i^* lies in the i-th subinterval $[x_{i-1}, x_i]$. Then the **definite integral of f from a to b** is
>
> $$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x,$$
>
> provided this limit exists and gives the same value for all possible choices of sample points. If it does exist, we say that f is **integrable** on $[a, b]$.
>
> [1] For example, we could choose our sample points to be right-hand endpoints, left-hand endpoints, midpoints, a combination of these, or any other sample points in the interval that we choose!

2. Consider again $f(x) = x^2 - 2x$ on the interval $[1, 3]$. Suppose that we are dividing the interval $[1, 3]$ into n subintervals. (Think about your answers to #1.)

   (a) What is the length of each subinterval? __________

   (b) What is the right-hand endpoint of the first subinterval? __________
   What is the height of the first right-hand rectangle? __________

   (c) What is the right-hand endpoint of the second subinterval? __________
   What is the height of the second right-hand rectangle? __________

   (d) What is the right-hand endpoint of the third subinterval? __________
   What is the height of the third right-hand rectangle? __________

   (e) What is the right-hand endpoint of the i^{th} rectangle? __________
   What is the height of the i-th right-hand rectangle? __________
   What is the **area** of the i-th right-hand rectangle? __________

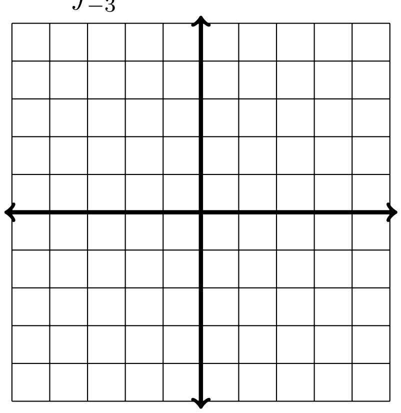
3. Using your answers to the previous problem, write down a limit that equals $\int_1^3 x^2 - 2x\,dx$.

4. Write down a limit that equals $\int_2^8 e^x\,dx$, using right-hand endpoints as your sample points.

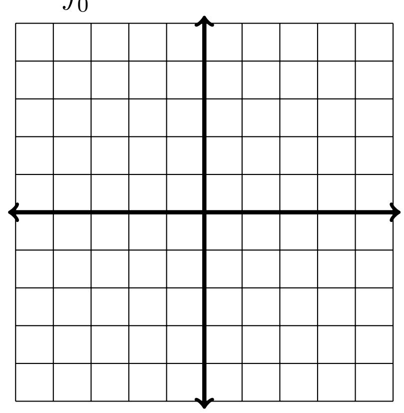
> *A definite integral represents the* **signed** *area under a curve (that is, the signed area between the curve and the x-axis). If a curve is above the x-axis that area is __________; if the curve is below the x-axis the area is __________.*

5. Evaluate the following definite integrals by drawing the function and interpreting the integral in terms of areas. Shade in the area you are computing with the integral.

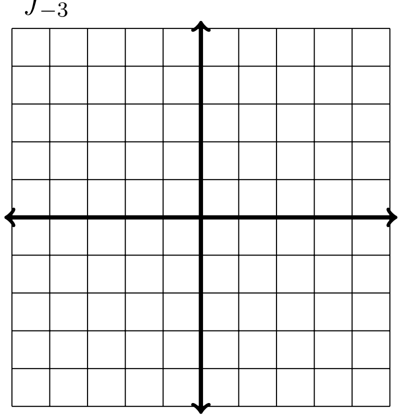
(a) $\int_{-3}^{3} (x-1)\,dx =$________



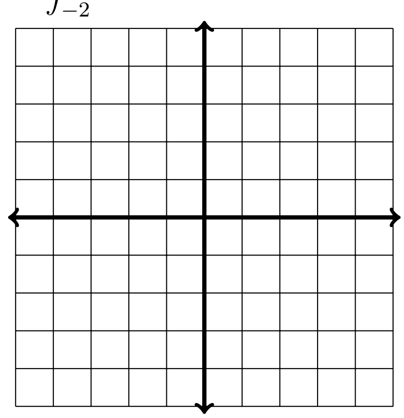
(b) $\int_0^4 \sqrt{16-x^2}\,dx=$ ________



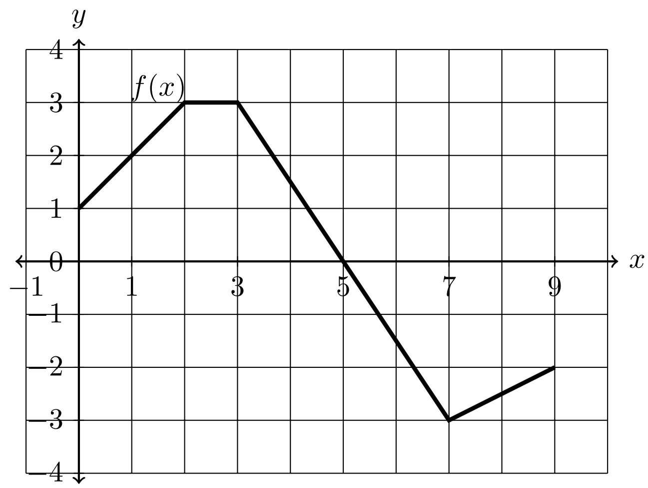
(c) $\int_{-3}^{3} (2+\sqrt{9-x^2})\,dx =$ ________



$\int_{-2}^{3} (|x|-3)\,dx =$ ________



6. The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.



(a) $\int_2^5 f(x)dx =$

(b) $\int_5^9 f(x)dx =$

(c) $\int_3^7 f(x)dx =$

Properties of the Definite Integral:

- $\int_a^b f(x)\,dx =$ ______________________
- $\int_a^a f(x)\,dx =$ ______________________
- $\int_a^b c\,dx =$ ______________________
- $\int_a^b cf(x)\,dx =$ ______________________
- $\int_a^b [f(x) \pm g(x)]dx =$ ______________________
- $\int_a^b f(x) + \int_b^c f(x)\,dx =$ ______________________
- $\int_b^a f(x)\,dx =$ ______________________

7. Using the fact that $\int_0^1 x^2 dx = \frac{1}{3}$ and $\int_1^2 x^2\,dx = \frac{7}{3}$, evaluate the following using the properties of integrals.

(a) $\int_1^0 x^2\,dx$ (b) $\int_0^1 5x^2\,dx$ (c) $\int_0^1 (4+3x^2)\,dx$ (d) $\int_0^2 x^2\,dx.$