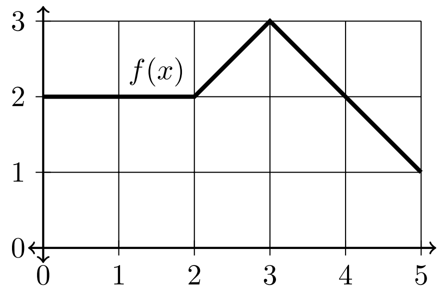
"Area So Far" functions

1. Let $f(x)$ be given by the graph below and define $A(x) = \int_0^x f(t)dt$.



Compute the following using the graph. Hint: $A(1) = \int_0^1 f(x)\,dx$, which calculates the area accumulated under the graph from $x = 0$ to $x = 1$.

$A(1) =$ $\int_0^1 f(x)\,dx = 2$ $\qquad f(1) =$ 2

$A(2) =$ $\int_0^2 f(x)\,dx = 4$ $\qquad f(2) =$ 2

$A(3) =$ $\int_0^3 f(x)\,dx = 7\frac{1}{2}$ $\qquad f(3) =$ 3

$A(4) =$ $\int_0^4 f(x)\,dx = 9$ $\qquad f(4) =$ 2

$A(5) =$ $\int_0^5 f(x)\,dx = 10\frac{1}{2}$ $\qquad f(5) =$ 1

The x-value in the interval $[0, 5]$ at which $A(x)$ attains its maximum is 5

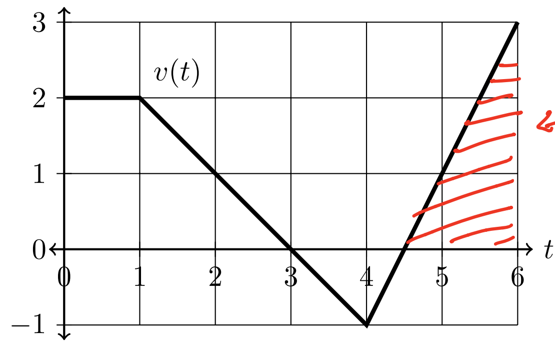
The maximum value of $A(x)$ on $[0, 5]$ is $10\frac{1}{2}$

The x-value in the interval $[0, 5]$ at which $f(x)$ attains its maximum is 3

The maximum value of $f(x)$ on $[0, 5]$ is 3

What can you say about the **rate of change** of $A(x)$? it is always positive

2. A toy car is travelling on a straight track. Its velocity $v(t)$, in meters per second, is given by the graph below. Define $s(t)$ to be the position of the car in meters, and suppose that $s(0) = 0$. Note that $s(t) = \int_0^t v(x)\,dx$. (Here, x is called the "dummy variable of integration".)



← This area is $\frac{1}{2}(3)(2) = 3$

Compute the following:

$s(2) =$ 3.5 $\qquad s(4) =$ 3 1/2 $\qquad s(6) =$ 3 + 3.5 = 6.5

$v(2) =$ 1 $\qquad v(4) =$ −1 $\qquad v(6) =$ 3

The t-value in the interval $[0, 6]$ at which $s(t)$ attains its maximum is 6

The maximum value of $s(t)$ on $[0, 6]$ is 6.5

The t-value in the interval $[0, 6]$ at which $s(t)$ attains its minimum is 0

The minimum value of $s(t)$ on $[0, 6]$ is 0

The t-value in the interval $[0, 6]$ at which ~~$s(t)$~~ v(t) attains its maximum is 6

The maximum value of ~~$s(t)$~~ v(t) on $[0, 6]$ is 3

The t-value in the interval $[0, 6]$ at which ~~$s(t)$~~ v(t) attains its minimum is 4

The minimum value of ~~$s(t)$~~ v(t) on $[0, 6]$ is −1

Describe the position of the car over the 6 seconds. Goes forward for 3 seconds, then goes backwards a little bit, for 1.5 seconds, then goes forward for 1.5 seconds

Describe the velocity of the car over the 6 seconds. Starts at 2 meters/s and stays at that speed for 1 s, then slows down to 0 for 2 s., then has negative velocity going to 0 again over next 1.5 second, then increases speed/goes faster for remaining 1.5 s.