§ 2.3

From Friday:

- Want to evaluate $\lim_{x\to a} f(x)$.

If $f(x)$ is made of pieces that are known to be simple and well-behaved (no big jumps or vertical asymptotes) you can take the limit of each piece separately & put them all back together.

[i.e. You can substitute in!]

obsessive example: $\lim_{x\to 2}\sqrt{3x^2+5} = \sqrt{3\cdot\left(\lim_{x\to 2} x\right)\left(\lim_{x\to 2} x\right)+\lim_{x\to 2} 5}$

$$= \sqrt{3\cdot 2\cdot 2 + 5} = \sqrt{17}$$

actually evaluating the limit.

practical example: $\lim_{x\to 2}\sqrt{3x^2+5} = \sqrt{3\cdot 2^2+5} = \sqrt{17}$

known functions

nothing bad happened

cautionary example:

$$\lim_{x\to 2}\frac{xe^x-2e^x}{x-2} = \frac{2e^2-2e^2}{2-2} = \frac{0}{0} \leftarrow \text{nonsense!}$$

plug in

Start over.
(Don't make stuff up!)

$$\lim_{x\to 2}\frac{\cancel{(x-2)}(e^x)}{\cancel{x-2}} = \lim_{x\to 2} e^x = e^2$$

Is this fair? Why?

SECTION 2-3 EXAMPLES

1. Evaluate each limit. Show your work or explain your reasoning.

   (a) $\lim_{x\to 8}(1+\sqrt[3]{x})(2-x^2)$ $= (1+\sqrt[3]{8})(2-8^2) = 3(-62) = -186$

   [Each "piece" is simple. Nothing is undefined.]

   (b) $\lim_{x\to 4}\frac{x^2-4x}{x^2-x-12} = \lim_{x\to 4}\frac{x(x-4)}{(x-4)(x+3)} = \lim_{x\to 4}\frac{x}{x+3} = \frac{4}{4+3} = \frac{4}{7}$

   (We get $\frac{0}{0}$ if we plug in.
   So, must do some algebra)

   (c) $\lim_{x\to 4}\frac{x^2}{x^2-x-12} = \lim_{x\to 4}\frac{x^2}{(x-4)(x+3)} = \text{DNE}$

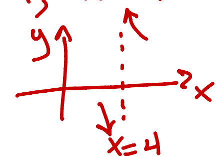
   [When you plug in, you get $\frac{16}{0}$. So, expect an infinite limit.]

   As $x\to 4^+$, $(x-4)(x+3)\to 0^+$

   As $x\to 4^-$, $(x-4)(x+3)\to 0^-$

   As $x\to 4$, $x^2\to 16>0$, always.

   Picture



   (d) $\lim_{x\to -3}\frac{\frac{1}{3}+\frac{1}{x}}{x+3} = \lim_{x\to -3}\left(\frac{1}{x+3}\right)\left(\frac{1}{3}+\frac{1}{x}\right) = \lim_{x\to -3}\left(\frac{1}{x+3}\right)\left(\frac{3+x}{3x}\right) = \lim_{x\to -3}\frac{1}{3x} = -\frac{1}{9}$

   Get $\frac{0}{0}$. So, algebra.

(e) $\lim_{x\to 0^-} \frac{|x|}{x}$ $= \lim_{x\to 0^-} \frac{-x}{x} = \lim_{x\to 0^-} -1 = -1$

$\boxed{\text{Since } x\to 0^-,\ x<0. \text{ So } |x| = -x.}$

(f) $\lim_{x\to 0} \frac{|x|}{x}$ $=$ DNE

From (e) we know $\lim_{x\to 0^-} \frac{|x|}{x} = -1$.

But if $x\to 0^+$, $|x| = x$. So, $\lim_{x\to 0^+} \frac{|x|}{x} = \lim_{x\to 0^+} \frac{x}{x} = 1$

(g) $\lim_{x\to 5^-} \frac{3x-15}{|5-x|}$ $= \lim_{x\to 5^-} \frac{3(x-5)}{|x-5|} = \lim_{x\to 5^-} \frac{3(x-5)}{-(x-5)} = \lim_{x\to 5^-} -3 = -3$

just algebra

as $x\to 5^-$, $x-5<0$. So $|x-5| = -(x-5)$.

(h) $\lim_{x\to \pi} \frac{2x}{\tan^2 x}$ $= +\infty$

$\boxed{\text{as } x\to\pi,\ 2x>0 \text{ and } \tan^2 x\to 0^+.}$