# SECTION 4.4 INDETERMINATE FORMS AND L'HOSPITAL'S RULE

WARM UP: Consider the limit $\lim_{x\to 2} \dfrac{x^2-4}{x^2-5x+6}$.

If you plug in 2 for $x$, what does the limit "look like"? ____________________

Evaluate the limit, using algebraic techniques from Chapter 2, and justifying each step.

$$\lim_{x\to 2} \frac{x^2-4}{x^2-5x+6} =$$

A limit is of *indeterminate type* (or 'type' for short) if we can't just "plug in $a$" to find the limit, or if different ways to write the same expression produce different limits! We describe indeterminate types by evaluating the limit of "pieces" by "plugging in $a$" and writing the resulting symbols, for example, $\frac{\infty}{\infty}$, $\frac{0}{0}$, or $\infty - \infty$. We can use *L'Hopital's rule* to help evaluate certain limits of indeterminate type.

**L'Hospital's Rule:** If a limit has the form (indeterminate type) of ________ or ________ then

$$\lim_{x\to a} \frac{f(x)}{g(x)} \overset{L'H}{=}$$

*provided*

1. Determine whether or not l'Hospital's Rule applies to the following examples, and if it does, apply it, and determine the limit.

   a) $\lim_{x\to 2} \dfrac{x^2-4}{x^2-5x+6}$ (type ______ )

   b) $\lim_{x\to 0} \dfrac{\sin x}{x}$ (type ______ )

QUESTION Why does l'Hospital's Rule work?

*Evaluate the following limits. Use L'Hopital's rule only when necessary!*

2. $\lim_{x \to 0} \frac{\tan(5x)}{\sin(3x)}$ (type ______ )

4. $\lim_{x \to 0} \frac{\cos(4x)}{e^{2x}}$ (type ______ )

3. $\lim_{u \to \infty} \frac{e^{u/10}}{u^2}$ (type ______ )

5. $\lim_{x \to 0} \frac{xe^x}{2^x - 1}$ (type ______ )

**Trickier applications of L'Hopital's rule**

| **Indeterminate form** | technique | **NOT indeterminate forms** | limit |
|---|---|---|---|
| $\frac{0}{0}$ | Algebra; L'H if necessary | $\infty + \infty$ | |
| $\frac{\infty}{\infty}$ | Algebra; L'H if necessary | $1^0$ | |
| $\infty - \infty$ | algebra to rewrite as $\frac{0}{0}$ or $\frac{\infty}{\infty}$ | $\frac{1}{\infty}$ | |
| $0 \cdot \infty$ | algebra to rewrite as $\frac{0}{0}$ | $\infty \cdot \infty$ and $\infty^\infty$ | |
| $1^\infty$ | Use logs to transform | $\frac{1}{0}$ | |
| $0^0$ | Use logs to transform | $0^\infty$ | |
| $\infty^0$ | Use logs to transform | $\infty^\infty$ | |

*Transform the following expressions into a form where you can use L'Hopital's rule to evaluate the limit, and then evaluate the limit.*

6. $\lim_{x\to 1^+} \left(\ln(x^4 - 1) - \ln(x^9 - 1)\right)$ (type ____________ )

7. $\lim_{x\to\infty} \sqrt{x}e^{-x/2}$ (type ____________ )

**Using logarithms to deal with limits of the form $1^\infty$ or $0^0$ or $\infty^0$**

8. Simplify the expressions below:

(a) If $y = a^b$, then $\ln y =$ ____________________

(b) If $\lim_{x\to a} \ln[f(x)] = L$, then $\lim_{x\to a} f(x) =$ ____________________

9. Now find the limit of the functions below by first taking the natural logarithm of the expression in the limit (like part(a) above). Then evaluate the limit of this transformed expression. Finally, use the answer of the transformed expression to obtain the limit of the original expression (like part (b) above).

(a) $\lim_{x\to\infty} x^{2/x}$ (type ______ )

(b) $\lim_{x\to 0^+} (1+\sin(2x))^{1/x}$ (type ______ )