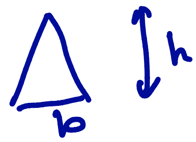
1. A Framework for Approaching Optimization

(a) Read the problem two or three times. Draw pictures. Label them. Pick specific numerical examples, to make the problem concrete. Be creative. Try more than just one approach.

use a picture to define variables



(b) Identify the quantity to be minimized or maximized (and which one... min or max).

maximize area

minimize distance

(c) Chose notation and explain what it means.

A - area

D - distance

x = # widgets sold

(d) Write the thing you want to maximize or minimize **as a function of one variable**, including a reasonable **domain**.

$C(x) = 1000 + 3x^2 + \frac{1000}{x^2}$ domain $(0, \infty)$

$D(x) = x + \sin(x)$ domain $[0, 10]$

(e) Use calculus to answer the question and justify that your answer is correct.

- Take derivative
- Find crit. pts.
- Determine which (if any) c.p's are associated w/ your problem
- Justify your choice
- Answer the question.

2. Why does *justification* matter?

Example: f(x) has c.p $x = 9.73$. $\boxed{\text{Ans: } x = 9.73.}$

What is wrong here?

What if $x = 9.73$ corresponds to a minimum and you were looking to maximize f(x)?

3. Find two positive numbers whose sum is 110 and whose product is a maximum.

(a) thinking: $x=10,\ y=100 \quad xy=1000$

$x=20,\ y=90 \quad xy=1800$

⋮

(b) maximize product $P=xy$

(c) Let x, y be the two positive numbers.

(d) Write P as a function of 1 variable.

Use $x+y=110$. So $y=110-x$.

So $P(x)=x(110-x)=110x-x^2$

$\boxed{\text{with domain } (0, 110)}$

Don't forget!

(e) Calculus

$P'(x)=110-2x=0$

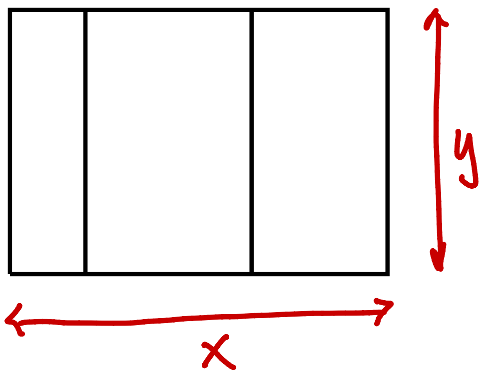
So $x=55$ (crit. pt.)

Justification: P is a parabola that opens down. Thus P has a max @ $x=55$

Answer: The two numbers are $x=y=55$.

4. A rancher has 800 feet of fencing with which to enclose three adjacent rectangular corrals. See figure below. What dimensions should be used so that the enclosed area will be a maximum?

a.



b. goal: maximize area

c. A - area
x - lenthy
y - height

d. Write area as a function of ONE variable:

$A = xy = \boxed{(400-2y)y}$

$= 400y - 2y^2$

$\boxed{A(y) = 400y - 2y^2}$

on $(0, 200)$

use $800 = 2x + 4y$

So $x = \dfrac{800-4y}{2} = 400 - 2y$

e. Calculus

$A'(y) = 400 - 4y = 0$. So $y = 100$. (crit. point)

Justification

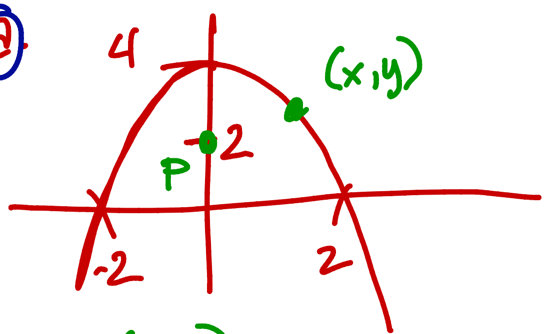
A is a downward opening parabola. So A(y) has a max at $y=100$.

Answer: The dimensions that maximize area are $y = 100$ ft and $x = 200$ ft.

$4 - \frac{3}{2} = \frac{8-3}{2} = 5/2$

5. Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 2)$? (Get started on this problem and once you have a function – that is, you have made it through part (d) of the Framework – look at the hint at the bottom of the page.)

(a)



P(0,2)

(b.) minimize distance from P to (x,y) on parabola

(c) D - distance from P to (x,y) on parabola.

(d) Want D as a function of x or y.

Use distance formula.

$$D = \sqrt{(x-0)^2 + (y-2)^2}$$

$\boxed{\text{Use } y = 4 - x^2}$

using symmetry.

$$D(x) = \sqrt{x^2 + (4-x^2-2)^2} = \sqrt{x^2 + (2-x^2)^2} \text{ on } [0,\infty)$$

* Because of HWT, I change D from distance to dis-ance squared *

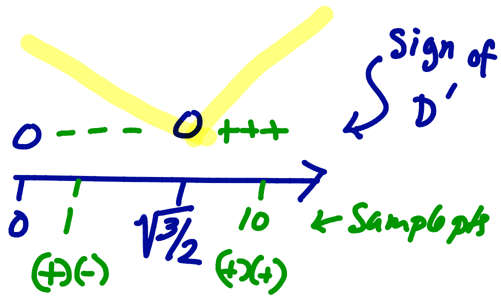
$$D(x) = x^2 + (2-x^2)^2 = 4 - 3x^2 + x^4$$

(e) Calculus

$$D'(x) = -6x + 4x^3 = 2x(-3 + 2x^2)$$

crit.pts: $x = 0$, $x = \pm\sqrt{3/2}$

local min @ $x = +\sqrt{3/2}$ by first derivative test. Because $x = \sqrt{3/2}$ is the unique c.p. on $[0,\infty)$ it is absolute (not just local).



Answer: The points on the parabola closest to (2,0) are $(\sqrt{3/2}, 5/2)$ and $(-\sqrt{\frac{3}{2}}, 5/2)$

HINT: Whenever you are asked to maximize or minimize distance, it is nearly ALWAYS easier to maximize or minimize the *square* of the distance. Why?