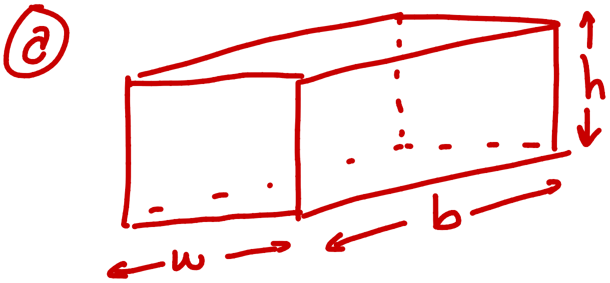
1. A rectangular storage container with lid is to have a volume of 36 cubic inches. the length of the base is three times the width. Material for the base and lid costs \$4 per square inch. Material for the sides costs \$1 per square inch. Find the cost of materials for the least expensive container.



b) minimize cost.

d) Want Cost as a function of w, b or h...

$$C = 4(2wb) + 1(2wh + 2bh)$$
$$= 8wb + 2wh + 2bh$$

$b = 3w$

$36 = wbh = w(3w)h$

$h = 12w^{-2}$

$$C(w) = 8w(3w) + 2w(12w^{-2}) + 2(3w)(12w^{-2})$$
$$= 24w^2 + 24w^{-1} + 3(24)w^{-1}$$
$$= 24w^2 + 96w^{-1} = 24(w^2 + 4w^{-1})$$

$\boxed{\text{on } (0, \infty)}$

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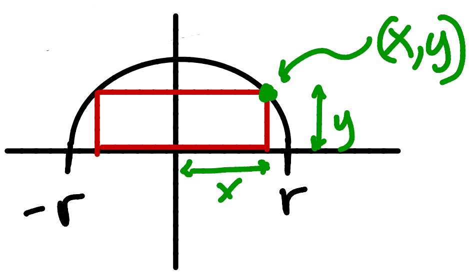
2. Find the area of the largest rectangle that can be inscribed in a semicircle of radius r



- maximize area
- $A = 2xy$

Semi-circle: $x^2 + y^2 = r^2 \rightarrow y = \sqrt{r^2 - x^2}$

So $A(x) = 2x(r^2 - x^2)^{1/2}$. Domain: $[0, 10]$

$r^2 - \frac{r^2}{2}$
$= \frac{r^2}{2}$

$$-\ A'(x) = 2(r^2 - x^2)^{1/2} + 2x\left(\tfrac{1}{2}\right)(r^2 - x^2)^{-1/2}(-2x)$$
$$= 2(r^2 - x^2)^{1/2} - \frac{2x^2}{(r^2 - x^2)^{1/2}} = 0$$

$$2(r^2 - x^2) = 2x^2$$
$$r^2 = 2x^2$$

$\sqrt{\frac{r^2}{2}} = \frac{r}{\sqrt{2}}$ $\quad x = \frac{r}{\sqrt{2}}$

Table

x	0	r	$r/\sqrt{2}$
$A(x)$	0	0	r^2

Answer:

The maximum area is r^2

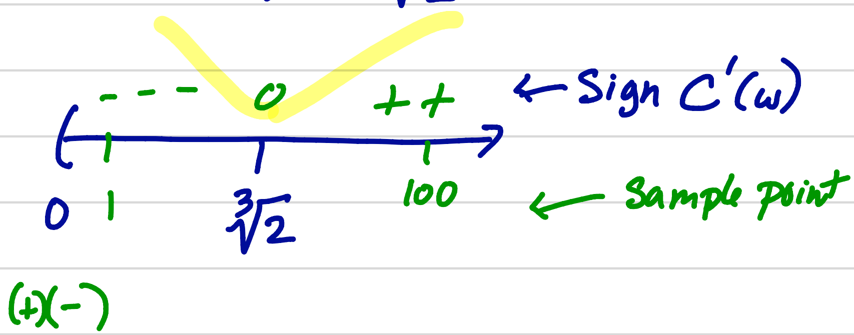
$$C(w) = 24\left(w^2 + 4w^{-1}\right)$$

$$C'(w) = 24\left(2w - 4w^{-2}\right) = 0$$

$$2w = \frac{4}{w^2}$$

$$w^3 = 2$$

$$w = \sqrt[3]{2}$$ ← critical point



Justification: By First Derivative Test, $C(w)$ has a local min @ $w = \sqrt[3]{2}$.
Since there is only 1 crit. pt., it corresponds to an absolute min.

$4 = 2^2 = 2^{\frac{6}{3}}$

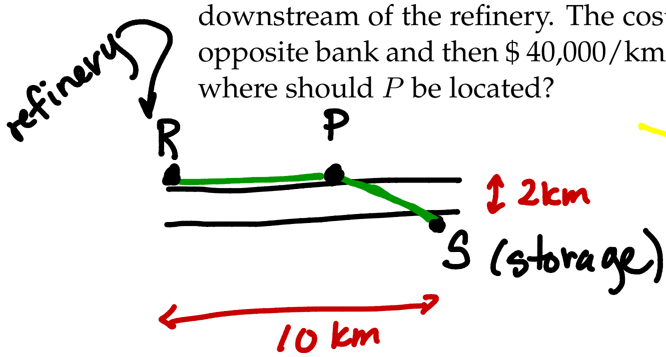
ANS: The minimum cost is

$$C(\sqrt[3]{2}) = 24\left(2^{2/3} + \frac{4}{2^{1/3}}\right) = 24\left(2^{2/3} + 2^{5/3}\right) \text{ dollars}$$

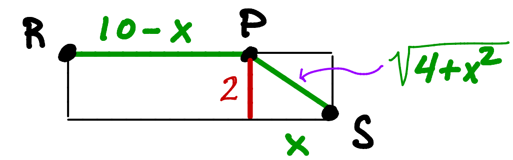
$$= 24(2^{2/3})(1+2)$$

$$= 72(2^{2/3})$$

3. An oil refinery is located on the north bank of a straight river that is 2 km wide. A pipeline is to be constructed from the refinery to storage tanks located on the south bank of the river 10 km downstream of the refinery. The cost of laying pipe is \$ 10,000/ km over land to a point P on the opposite bank and then \$ 40,000/km under the river to the tanks. To minimize the cost of pipeline, where should P be located?



redraw



minimize cost

– C = cost in \$10,000.

$C(x) = 1(10-x) + 4(4+x^2)^{1/2}$

domain $[0, 10]$

– $C'(x) = -1 + 2(4+x^2)^{-1/2}(2x)$

$= -1 + \dfrac{4x}{\sqrt{4+x^2}} = 0$

$4x = \sqrt{4+x^2}$

$16x^2 = 4 + x^2$

$x^2 = 4/15$

$x = 2/\sqrt{15}$

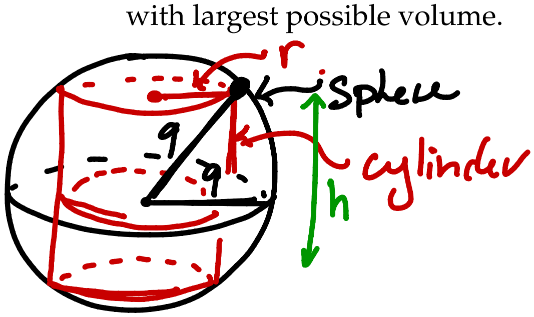
table

x	0	10	$\frac{2}{\sqrt{15}}$
C(x)	18	$8\sqrt{104}$	10.48

C_{min}.

ANS: P should be $10 - \frac{2}{\sqrt{15}} \approx 9.5$ km down river.

4. A right circular cylinder is inscribed in a sphere of radius 9. Find the dimensions of the cylinder with largest possible volume.



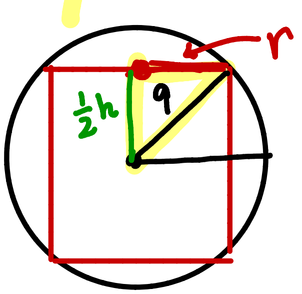
– maximize volume of cylinder

– $V = \pi r^2 \boxed{h}$

$V(h) = \pi\left(81 - \frac{h^2}{4}\right)h = 81\pi h - \frac{\pi}{4}h^3$

domain $[0, 18]$

redraw picture of cross-section



$r^2 + \frac{h^2}{4} = 81$

$r^2 = 81 - \frac{h^2}{4}$

$V'(h) = 81\pi - \frac{3\pi}{4}h^2 = 0$

$h^2 = 4 \cdot 27$

$h = 6\sqrt{3}$ ← only crit. pt.

$V(0) = V(18) = 0$. So V has a max @ $x = 6\sqrt{3}$

– Answer: Volume is maximized when $h = 6\sqrt{3}$ and $r = \sqrt{81 - \frac{36 \cdot 3}{4}} = 3\sqrt{6}$

SECTION 4.7 APPLIED OPTIMIZATION (DAY 2)

3. An oil refinery is located on the north bank of a straight river that is 2 km wide. A pipeline is to be constructed from the refinery to storage tanks located on the south bank of the river 10 km downstream of the refinery. The cost of laying pipe is \$ 10,000/ km over land to a point P on the opposite bank and then \$ 40,000/km under the river to the tanks. To minimize the cost of pipeline, where should P be located?

4. A right circular cylinder is inscribed in a sphere of radius r. Find the dimensions of the cylinder with largest possible volume.