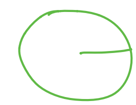
1. Find all critical points of the function $f(x) = \sin(x)^{1/3}$.

$$f'(x) = \frac{1}{3}\sin(x)^{-2/3}(\cos(x)) = \frac{\cos(x)}{3(\sin(x))^{2/3}}$$

$f'(x)$ is undefined where $\sin(x) = 0 \Rightarrow k\pi$ for k an integer



$f'(x) = 0$ where $\cos(x) = 0 \Rightarrow x = \frac{\pi}{2} + 2k\pi$ or $x = -\frac{\pi}{2} + 2k\pi$ for k an integer

That is, there are infinitely many critical points...

2. Find the absolute maximum and minimum values (y-values) of $f(x) = e^{-x^2}$ on the interval $[-2, 3]$, and the locations (x-values) where those values are attained.

Critical points: $f'(x) = e^{-x^2}(-2x) = -\frac{2x}{e^{x^2}}$

Note $e^{x^2} > 0$ for all x, so only critical points are where $f'(x) = 0 \Rightarrow e^{-x^2}(-2x) = 0 \Rightarrow x = 0$

x	$f(x)$	
-2	$e^{-2^2} = \frac{1}{e^4}$	
0	$e^{-0^2} = e^0 = 1$	← $y = 1$ is Absolute max, at $x = 0$
3	$e^{-3^2} = \frac{1}{e^9}$	← $y = 1/e^9$ is Absolute min, at $x = 3$

3. A ball thrown in the air at time $t = 0$ has a height given by

$$h(t) = h_0 + v_0 t - \frac{1}{2} g_0 t^2$$

meters where t is measured in seconds, h_0 is the height at time 0, v_0 is the velocity (in meters per second) at time 0 and g_0 is the constant acceleration due to gravity (in $\mathrm{m/s^2}$). Assuming $v_0 > 0$, find the time that the ball attains its maximum height. Then find the maximum height.

$h'(t) = v_0 - \frac{1}{2} g_0 (2t)$

So $h'(t) = 0 \Rightarrow v_0 - \frac{1}{2} g_0 (2t) = 0 \Rightarrow g_0 t = v_0 \Rightarrow$

$t = \frac{v_0}{g_0}$ is the only critical point.

At $t=0$, height $= h_0$. Observe absolute min is where $h=0$.

Evaluate at $\frac{v_0}{g_0}$: $h\left(\frac{v_0}{g_0}\right) = h_0 + \frac{v_0^2}{g_0} - \frac{1}{2} \cdot g_0 \left(\frac{v_0}{g_0}\right)^2$

$= h_0 + \frac{v_0^2}{g_0} - \frac{v_0^2}{2g_0} = h_0 + \frac{2v_0^2 - v_0^2}{2g_0} = \boxed{h_0 + \frac{v_0^2}{2g_0}}$

① $h(t)$ attains its maximum height at $t = \frac{v_0}{g_0}$ seconds

② The maximum height attained is $h(t) = h_0 + \frac{v_0^2}{2g_0}$ meters