SECTION 4-3: HOW DERIVATIVES AFFECT THE SHAPE OF A GRAPH, DAY 1

1. Consider $f(x) = \frac{2}{3}x^3 + x^2 - 12x + 7$, and observe $f'(x) = 2x^2 - 2x - 12 = 2(x-2)(x+3)$.

(a) What are the critical points of $f(x)$? (Where does $f'(x) = 0$?) ____________________

(b) Fill in the following table, by evaluating $f'(x)$ at "sample points" in the intervals:

x	$x < -3$	-3	$-3 < x < 2$	2	$x > 2$
sample point	-4	-3	0	2	5
sign or value of f'					
Increasing/decreasing: f is ↗ or ↘					

(c) On what interval(s) is $f(x)$ increasing? ______________ decreasing? ______________

(d) Use the First Derivative Test to determine where f has a local max and local min (if any):

i. Local max at $x =$ ____ because f' goes from __ to __.

ii. Local min at $x =$ ____ because f' goes from __ to __

(e) It is a fact that $f''(x) = 4x - 2$, so $f''(x) = 0$ when $x =$________.
Fill in the expanded chart:

x	$x < -3$	-3	$-3 < x < 1/2$	$1/2$	$1/2 < x < 2$	2	$x > 2$
sample point	-4	-3	0	$1/2$	1	2	5
sign or value of f'							
sign or value of f''							
concavity: f is ↗↘↗↘							

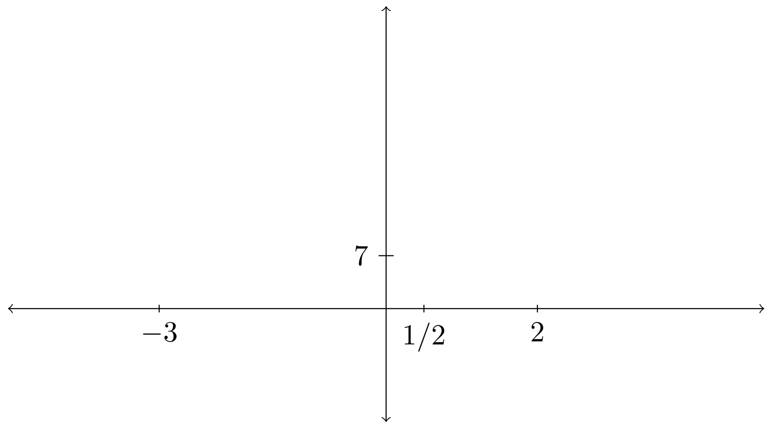
(f) Use the Second Derivative Test to determine where f has local maxima or minima:

i. Local max at $x =$ ____ because $f'(\_\_) =$ __ and $f''(\_\_)$ __________.

ii. Local max at $x =$ ____ because $f'(\_\_) =$ __ and $f''(\_\_)$ __________.

(g) Where does f have an inflection point? $x =$____
How do you know? ______________________________

(h) Use the information you collected to sketch the graph of $f(x)$. You don't have to be accurate with the y-values, but they should be correct relative to each other. Because $f(0) = 7$, you can use that to "nail down" the position of your curve on the graph. Note that



2. Consider $g(x) = xe^x$, and note $g'(x) = xe^x + x = e^x(x+1)$ and $g''(x) = e^x(x+2)$.

(a) What are the critical point(s) of $g(x)$?

(b) Where is g increasing?

(c) Use the First Derivative Test to determine whether g has a local max or min at its critical point.

(d) Use the Second Derivative Test to determine whether g has a local max or min at its critical point.

3. Consider the function $h(x) = x^3$ and observe $h'(x) = 3x^2$ and $h''(x) = 6x$.

   (a) What are the critical point(s) of $h(x)$?

   (b) What happens when you try to use the Second Derivative Test to determine whether h has a local max or min at its critical point?

   (c) Make a table of first and second derivatives to determine where h is increasing, decreasing, concave up, and/or concave down. Then sketch h.

4. Consider the function $j(x) = x^4$ and observe $j'(x) = 4x^3$ and $j''(x) = 12x^2$.

   (a) What are the critical point(s) of $j(x)$?

   (b) What happens when you try to use the Second Derivative Test to determine whether j has a local max or min at its critical point?

   (c) Make a table of first and second derivatives to determine where j is increasing, decreasing, concave up, and/or concave down. Then sketch j.