2-7 EXAMPLES

1. Write two expressions for the slope of the tangent line to the function $f(x)$ at $x = a$. (Hint: Both should involve a limit. One has $x \to a$ and one has $h \to 0$. Try to produce the expressions by thinking about where they come from.)

$$m_{tan} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

For all of the problems on this worksheet, you need to use one of the expressions above. You should NOT use a short-cut rule we have not yet covered.

2. For each function and a-value below, i) find the slope of the tangent line to $f(x)$ at $x = a$ and (ii) write the equation of the line tangent to $f(x)$ at $x = a$.

   (a) $f(x) = \sqrt{2x}$ when $a = 8$.

(i) $$m = \lim_{h \to 0} \frac{\sqrt{2(8+h)} - \sqrt{2 \cdot 8}}{h} = \lim_{h \to 0} \frac{\sqrt{16+2h} - 4}{h} \cdot \frac{\sqrt{16+2h} + 4}{\sqrt{16+2h} + 4}$$

$$= \lim_{h \to 0} \frac{16 + 2h - 16}{h(\sqrt{16+2h} + 4)} = \lim_{h \to 0} \frac{2h}{h(\sqrt{16+2h} + 4)} = \lim_{h \to 0} \frac{2}{\sqrt{16+2h} + 4} = \frac{2}{8} = \frac{1}{4}$$

(ii) $m = \frac{1}{4}$, point $P = (8, 4)$

line: $y - 4 = \frac{1}{4}(x - 8)$ or $y = \frac{1}{4}x + 2$

   (b) $f(x) = \frac{3}{2-x}$ when $a = 1$.

$f(a+h)$ $f(a)$

(i) $$m = \lim_{h \to 0} \frac{1}{h}\left[\frac{3}{2-(1+h)} - \frac{3}{2-1}\right] = \lim_{h \to 0} \frac{1}{h}\left[\frac{3}{1-h} - \frac{3}{1}\right] = \lim_{h \to 0} \frac{1}{h}\left[\frac{3 - 3(1-h)}{1-h}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{3h}{1-h}\right] = \lim_{h \to 0} \frac{3}{1-h} = 3$$

(ii) $m = 3$; point: $(1, 3)$

line: $y - 3 = 3(x - 1)$ or $y = 3x$

3. (a) For $f(x) = 2x - x^2$, find $f'(a)$.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{2(a+h) - (a+h)^2 - [2a - a^2]}{h}$$

$$= \lim_{h \to 0} \frac{2a + 2h - a^2 - 2ah - h^2 - 2a + a^2}{h} = \lim_{h \to 0} \frac{2h - 2ah - h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2 - 2a - h)}{h} = \lim_{h \to 0} 2 - 2a - h = 2 - 2a$$

answer: $f'(a) = 2 - 2a$

(b) Find $f'(0)$, $f'(1)$, $f'(2)$ and $f'(3)$. (You could just make a table of values...)

x	0	1	2	3
$f'(x)$	2	0	-2	-4

(c) Do your answers to part (b) *seem* reasonable? Why or why not?

$f(x) = 2x - x^2 = x(2-x)$

Do these seem reasonable?
Well, yes. The numbers fit with what I know the graph looks like...