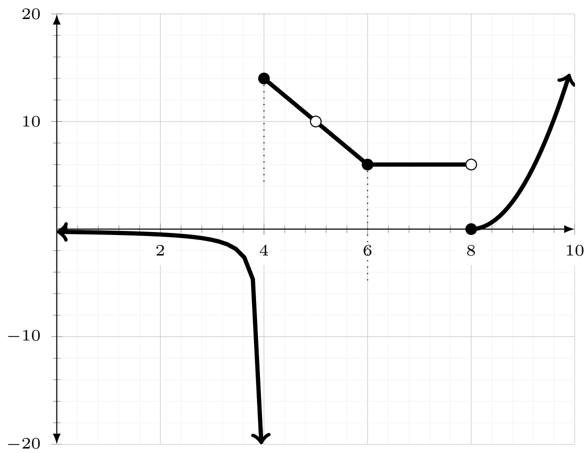
Lecture Notes: Chapters 1 & 2 Review

Practice Problems:

1. Use the graph of $f(x)$ below to answer the following questions.



(a) Assuming the arrows on the graph indicate a continued curve in that direction, make an educated guess at the domain of the function $f(x)$.

$(-\infty, 5) \cup (5, \infty)$

(b) Find all x-values in the domain of $f(x)$ for which $f(x)$

i. fails to be continuous.

$x = 4, 8$

ii. fails to be differentiable.

$x = 4, 6, 8$

(c) Evaluate the following limits or explain why they do not exist.

(i) $\lim_{x\to 4^-} f(x) =$ $-\infty$

(ii) $\lim_{x\to 4^+} f(x) =$ 14

(iii) $\lim_{x\to 4} f(x) =$ DNE

(iv) $\lim_{x\to 5} f(x) =$ 10

(v) $\lim_{x\to 6} f(x) =$ 6

(vi) $\lim_{x\to 7} f(x) =$ 6

(vi) $\lim_{x\to 8} f(x) =$ DNE

(vii) $\lim_{x\to 8^-} f(x) =$ 6

2. Find the horizontal and vertical asymptotes (if any) of the graph of $f(x) = \frac{2x^2}{3x^2+2x-1}$ and show your answers are correct.

$\lim_{x\to\infty} \frac{2x^2}{3x^2+2x-1} = \frac{2}{3}$. So $y = 2/3$ is a horizontal asymptote.

(Note $\lim_{x\to -\infty} f(x) = 2/3$, too.)

$3x^2+2x-1 = (3x-1)(x+1)$

$\lim_{x\to \frac{1}{3}^+} \frac{2x^2}{(3x-1)(x+1)} = +\infty$ and $\lim_{x\to -1^+} \frac{2x^2}{(3x-1)(x+1)} = -\infty$.

So $x = \frac{1}{3}$ and $x = -1$ are asymptotes.

Vertical.

3. Evaluate the following limits. Show your work. *Make sure you are writing your mathematics correctly and clearly.*

(a) $\lim_{t\to 2}\left(\frac{t^2-4}{t^3-3t+5}\right)^3 = \left(\frac{0}{7}\right)^3 = 0$

$8+5-6$
$=7$

(b) $\lim_{x\to 4^-}\frac{x^2+3x}{x^2-x-12} = \lim_{x\to 4^-}\frac{x^2+3x}{(x-4)(x+3)} = -\infty$

as $x\to 4^-$, $x^2+3x \to 16+12=28$, $x+3\to 7$

and $x-4\to 0^-$. That is $x-4$ is always negative.

∗(c) $\lim_{x\to 4}\frac{x^2-4x}{x^2-x-12} = \lim_{x\to 4}\frac{x(x-4)}{(x-4)(x+3)} = \lim_{x\to 4}\frac{x}{x+3} = \frac{4}{7}$

(d) $\lim_{h\to 0}\frac{(h-5)^2-25}{h} = \lim_{h\to 0}\frac{h^2-10h+25-25}{h} = \lim_{h\to 0} h-10$

$= 10$

4. For each function below, determine all the values in the domain of the function for which the function is continuous.

(a) $f(x) = \begin{cases} \frac{3}{x+5} & x<1 \\ \frac{x+1}{2} & 1\le x\le 3 \\ x^2-7 & 3<x \end{cases}$

as $x\to 1^-$, $f(x)\to \frac{3}{6}=\frac{1}{2}$. as $x\to 1^+$, $f(x)\to\frac{2}{2}=1$. So $\lim_{x\to 1} f(x)$ does not exist.

as $x\to 3^-$, $f(x)\to\frac{4}{2}=2$. as $x\to 3^+$, $f(x)\to 3^2-7=9-7=2$. So $\lim_{x\to 3} f(x)=2=f(3)$.

ANS: $f(x)$ is continuoas for $(-\infty,1)\cup(1,\infty)$

(b) $g(x) = \frac{2^x+1}{\sqrt{1-x}}$. This function will be continuous where it is defined because it's built of continuous functions.

We need $1-x>0$. So $1>x$.

Ans: $(-\infty, 1)$

5. Find the limit or show that it does not exist.

(a) $\lim\limits_{x\to-\infty} \frac{2-x}{3x^2-x} = \lim\limits_{x\to-\infty} \frac{\frac{2}{x^2}-\frac{1}{x}}{3-\frac{1}{x}} = \frac{0}{3} = 0$

(b) $\lim\limits_{x\to\infty} [\ln(1+x^2) - \ln(1+x)] = \lim\limits_{x\to\infty} \ln\left(\frac{1+x^2}{1+x}\right) = \infty.$

Since $\lim\limits_{x\to\infty} \frac{1+x^2}{1+x} = \lim\limits_{x\to\infty} \frac{\frac{1}{x}+x}{\frac{1}{x}+1} = \infty.$

(c) $\lim\limits_{x\to\infty} \frac{3x^2+2x}{\sqrt{x^4+2x}} \cdot \frac{1/x^2}{1/x^2} = \lim\limits_{x\to\infty} \frac{3+\frac{2}{x^2}}{\sqrt{1+\frac{2}{x^3}}} = \frac{3}{\sqrt{1}} = 3$

6. The displacement (in feet) of a particle moving in a straight line is given by $s(t) = 9t - t^2$ where t is measured in seconds.

(a) Find the average velocity from $t = 1$ to $t = 3$ and include units with your answer.

average vel. $= \dfrac{S(3)-S(1)}{3-1} = \dfrac{18-8}{2} = 5$ ft/s

$s(3) = 9 \cdot 3 - 3^2 = 18$

$s(1) = 9 - 1 = 8$

* Use the definition!!

(b) Find the instantaneous velocity of the particle when $t = 1$ and include units with your answer.

$$v(1) = \lim_{h \to 0} \frac{S(1+h) - S(1)}{h} = \lim_{h \to 0} \frac{[9(1+h) - (1+h)^2] - 8}{h}$$

$$= \lim_{h \to 0} \frac{9 + 9h - 1 - 2h - h^2 - 8}{h} = \lim_{h \to 0} \frac{7h - h^2}{h} = \lim_{h \to 0} 7 - h = 7$$

So velocity at $t=1$ is 7 ft/s