1. (Warm-up)

   (a) Find the derivative of $f(x) = (x^2+1)^{\sin x}$.

$y = (x^2+1)^{\sin x}$

$\ln y = (\sin x)\ln(x^2+1)$

$\frac{1}{y}\cdot y' = (\sin x)\left(\frac{2x}{x^2+1}\right) + (\cos x)\cdot \ln(x^2+1)$

So $y' = y\left(\frac{2x\sin x}{x^2+1} + \cos x \ln(x^2+1)\right)$

$y' = (x^2+1)^{\sin x}\left[\frac{2x\sin x}{x^2+1} + \cos x \ln(x^2+1)\right]$

   (b) Find the derivative of $f(x) = \cos\left(100a\,\frac{3-x}{\sqrt{2}}\right)$ *in the most efficient manner.*

$f'(x) = -\sin\left(\frac{100a}{\sqrt{2}}(3-x)\right)\cdot\frac{-100a}{\sqrt{2}} = \frac{100a}{\sqrt{2}}\sin\left(\frac{100a}{\sqrt{2}}(3-x)\right)$

2. Let $n = f(t)$ model the number of voles in my garden starting in year 2000 where n counts the number of voles and t is measured in years. Assume that in 2000, 5 voles lived in the garden and that I estimate that the number of voles doubles every three years.

   (a) Find $f(0)$, $f(3)$, $f(6)$, and $f(9)$ using the assumptions above. (include units)

$f(0) = 5$ voles

$f(3) = 10$ voles

$f(6) = 20$ voles

$f(9) = 40$ voles

pattern recognition

t	0	$3\cdot 1$	$3\cdot 2$	$3\cdot 3$
n	5	$5\cdot 2^1$	$5\cdot 2^2$	$5\cdot 2^3$

   (b) Find an expression for $n = f(t)$ in general.

$n = f(t) = 5\cdot 2^{t/3}$

   (c) Find and interpret $f'(10)$.

$f'(t) = 5\cdot(\ln 2)\cdot 2^{t/3}\cdot\frac{1}{3}$

$= \frac{5}{3}(\ln 2)\, 2^{t/3}$

$f'(10) = \frac{5}{3}(\ln 2)\, 2^{10/3}$

≈ 11.64

In 2010, the vole population of the garden is increasing at a rate of 11.6 voles per year.

3. The position of a particle moving along a straight line is given by: $s(t) = 3\sin(\pi t/2)$ for $t \geq 0$ where t is measured in seconds and s is measured in feet.

(a) Find the position at which the particle starts.

Find s when $t=0$.

$s(0) = 3\sin(0) = 0$

(b) Where is the particle 3 seconds after starting?

Find s when $t=3$.

$s(3) = 3\sin\left(\frac{3\pi}{2}\right) = -3$

(c) When is the particle in position 0?

Find t when $s=0$

$0 = 3\sin\left(\frac{\pi t}{2}\right) \to 0 = \sin\left(\frac{\pi t}{2}\right)$

So $\pi \cdot k = \frac{\pi t}{2}$

So $\underline{t = 2k,\ k \text{ integer}}$

(d) Find the velocity and the acceleration of the particle.

$s = 3\sin\left(\frac{\pi t}{2}\right)$

$s' = v = 3 \cdot \cos\left(\frac{\pi t}{2}\right) \cdot \frac{\pi}{2} = \frac{3\pi}{2}\cos\left(\frac{\pi t}{2}\right)$; $s'' = a = \frac{3\pi}{2} \cdot \left[-\cos\left(\frac{\pi t}{2}\right)\right] \cdot \frac{\pi}{2}$

$= -\frac{3\pi^2}{4}\cos\left(\frac{\pi t}{2}\right)$

(e) When is the particle at rest?

Find t when $v=0$.

$0 = \frac{3\pi}{2}\cos\left(\frac{\pi t}{2}\right)$. $\to$ So $0 = \cos\left(\frac{\pi}{2}t\right)$.

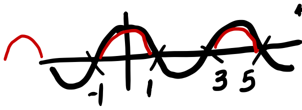
$\frac{(2k+1)\pi}{2} = \frac{\pi}{2}t$;

So $t = 2k+1$ for k integer.

(f) When is the particle moving in the positive direction?

Find t when $v > 0$.

$v = \frac{3}{2}\cos\left(\frac{\pi t}{2}\right)$



answer:

$\ldots (-5,-3) \cup (-1,1) \cup (3,5), (7,9), \ldots$

(g) Find the total distance the particle travels in the first 4 seconds.

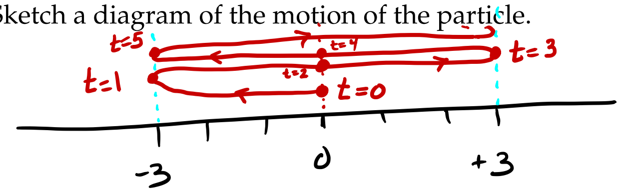
$s(0) = 0$

$s(1) = 3$

particle travels 3 feet in first second.

So (by symmetry) it travels 12 feet (4·3) in the first 4 seconds.

(h) Sketch a diagram of the motion of the particle.



4. Let $n = f(t)$ model the number of voles in my garden starting in year 2000 where n counts the number of voles and t is measured in years. Assume that in 2000, three voles lived in the garden and that I estimate that the number of voles doubles every three years.

   (a) Find $f(0)$, $f(3)$, $f(6)$, and $f(9)$ using the assumptions above. (include units)

   (b) Find an expression for $n = f(t)$ in general.

   (c) Find and interpret $f'(10)$.

5. The volume of a growing spherical cell is modeled by $V = \frac{4}{3}\pi r^3$ where r is the radius of the cell measured in micrometers ($1\mu m = 10^{-6}m$.)

   (a) Find and interpret $V(4)$. (include units)

   $V(4) = \frac{4}{3}\pi 4^3 = \frac{256}{3}\pi \approx 268\ \mu m^3$

   The cell has volume $268\mu m^3$ when the radius is $4\mu m$.

   (b) Find the average rate of change of the volume of the cell when its radius increases from 4 to 4.1 μm.

   $\frac{V(4.1)-V(4)}{0.1} = 206.13\ \mu m^3/\mu m$

   (c) Find the instantaneous rate of change of the volume with respect to radius when $r = 4\mu m$ and interpret your answer.

   $\frac{dV}{dr} = 4\pi r^2$; $\left.\frac{dV}{dr}\right|_{r=4} = 4\cdot\pi\cdot 4^2 \approx 201.06\ \mu m^3/\mu$

   At the instant the radius is $4\mu m$, the volume is increasing at a rate of 201 cubic micrometers per $1\mu m$ increase in the radius.

   (d) What familiar formula is given by dV/dr and can you give an intuitive explanation for why this is?

   Surface area of sphere $= 4\pi r^2$.

sphere

increase radius by a little and the resulting increase in volume is like adding a layer on the outside.. on the surface!