

Name: \_\_\_\_\_ / 12

Instructor: Bueler | Jurkowski | Maxwell

- There are 12 points possible on this proficiency: **One point per problem. No partial credit.**
- A passing score is 10/12.
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with**  $f'(x) =$ ,  $dy/dx =$ , or similar.
- Circle your final answer.

**Compute the derivatives of the following functions.**

1.  $f(x) = \pi x^2 - \frac{x - \sqrt{5}}{9}$

$$f'(x) = 2\pi x - \frac{1}{9}$$

2.  $y = x^3 \ln(x)$

$$\begin{aligned} y' &= 3x^2 \ln(x) + x^3 \frac{1}{x} \\ &= x^2 [3 \ln(x) + 1] \end{aligned}$$

3.  $y = \tan(1 + x^4)$

$$y' = \sec^2(1 + x^4) \cdot 4x^3$$

$$4. \ g(r) = \frac{\cos(r)}{1-r^2}$$

$$\begin{aligned} g'(r) &= \frac{-\sin(r)(1-r^2) - \cos(r)(-2r)}{(1-r^2)^2} \\ &= \boxed{\frac{2r\cos(r) - (1-r^2)\sin(r)}{(1-r^2)^2}} \end{aligned}$$

$$5. \ h(w) = \arctan(\sin(2w-9))$$

$$h'(w) = \boxed{\frac{1}{1 + (\sin(2w-9))^2} \cdot \cos(2w-9) \cdot 2}$$

$$6. \ f(t) = \sec(te^t)$$

$$\begin{aligned} f'(t) &= \sec(te^t) \tan(te^t) \cdot [1 \cdot e^t + t \cdot e^t] \\ &= \boxed{\sec(te^t) \tan(te^t) \cdot [1+t]e^t} \end{aligned}$$

7.  $f(r) = \ln(1 + r^k)$  where  $k$  is a fixed constant.

$$f'(r) = \frac{1}{1+r^k} \cdot kr^{k-1}$$

8.  $y = (1+x^2)e^{\sin(\pi x)}$

$$y' = 2x e^{\sin(\pi x)} + (1+x^2) e^{\sin(\pi x)} \cos(\pi x) \cdot \pi$$

9.  $y = \sqrt{x} \ln(x) \arcsin(x)$

$$y' = \frac{1}{2} x^{-1/2} \ln(x) \arcsin(x) + \sqrt{x} \left[ \frac{1}{x} \arcsin(x) + \sqrt{x} \ln(x) \right] \frac{1}{\sqrt{1-x^2}}$$

10.  $f(x) = \cos(x) \sin(1 - 2x^3)$

$$f'(x) = -\sin(x) \sin(1 - 2x^3) + \cos(x) \cos(1 - 2x^3) \cdot (-6x^2)$$

11.  $h(w) = \frac{1}{\sin(w)}$

$$h'(w) = -\frac{1}{\sin^2(w)} \cdot \frac{d}{dw} \sin(w)$$

$$= -\frac{\cos(w)}{\sin(w)} \cdot \frac{1}{\sin(w)} = -\cot(w) \csc(w)$$

12. Compute  $dy/dx$  if  $x\sin(y) + 3xy^2 = e^x$ . You must solve for  $dy/dx$ .

$$\sin(y) + x\cos(y)y' + 3y^2 + 6xyy' = e^x$$

$$[x\cos(y) + 6xy]y' = e^x - \sin(y) - 3y^2$$

$$y' = \frac{e^x - \sin(y) - 3y^2}{x\cos(y) + 6xy}$$