Name: Solutions ________ / 12

Instructor: **Bueler** | **Jurkowski** | **Maxwell**

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- For at least one problem you must indicate correct use of a constant of integration.
- Circle your final answer.

1. [12 points] Compute the following definite/indefinite integrals.

a. $\int 7\cos(x) + \pi^6 - \sqrt{x}\, dx$

$$\boxed{7\sin(x) + \pi^6 x - \frac{2}{3}x^{3/2} + C}$$

b. $\int \sec^2(7x)\, dx$

$u = 7x$

$du = 7dx$

$$\int \sec^2(7x)dx = \int \frac{1}{7}\sec^2(u)du$$

$$= \frac{1}{7}\tan(u) + C$$

$$= \boxed{\frac{1}{7}\tan(7x) + C}$$

c. $\int_0^5 t^3(1-t)\, dt$

$$\int_0^5 t^3 - t^4 = \left.\frac{t^4}{4} - \frac{t^5}{5}\right|_0^5 = \frac{5^4}{4} - \frac{5^5}{5} - \left(\frac{0}{4} - \frac{0}{5}\right)$$ ok answer!

$$= \frac{5^5 - 4\cdot 5^5}{20} = \frac{-3\cdot 5^5}{20} = \boxed{-\frac{3\cdot 5^4}{4}}$$

d. $\displaystyle\int \frac{x^2}{\sqrt{x^3+5}}\,dx$

$u = x^3+5$

$du = 3x^2\,dx$

$$\int \frac{1}{3}\,\frac{1}{\sqrt{u}}\,du = \int \frac{1}{3}\,u^{-1/2}\,du = \frac{1}{3}\cdot 2\cdot u^{1/2} + C$$

$$= \boxed{\frac{2}{3}\left(x^3+5\right)^{1/2} + C}$$

e. $\displaystyle\int v\sqrt{v-8}\,dv$

$u = v-8$

$du = dv$

$$\int (u+8)\sqrt{u}\,du = \int u^{3/2} + 8u^{1/2}\,du$$

$$= \frac{2}{5}u^{5/2} + 8\cdot\frac{2}{3}u^{3/2} + C$$

$$= \boxed{\frac{2}{5}(v-8)^{5/2} + \frac{16}{3}(v-8)^{3/2} + C}$$

f. $\displaystyle\int \frac{\sin(x)}{\cos(x)}\,dx$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$$\int -\frac{du}{u} = -\ln(|u|) + C$$

$$= -\ln(|\cos(x)|) + C$$

$$= \boxed{\ln(|\sec(x)|) + C}$$

g. $\displaystyle\int \frac{6}{\sqrt{1-x^2}}\,dx$

$$\boxed{6\arcsin(x)}$$

h. $\displaystyle\int e^t - t^3\cos(t^4)\,dt$

$$\int t^3\cos(t^4) = \int \frac{1}{4}\cos(u)\,du = \frac{1}{4}\sin(t^4) + C$$

$u = t^4$

$du = 4t^3\,dt$

$$\int e^t\,dt = e^t + C$$

$$\int e^t - t^3\cos(t^4)\,dt = \boxed{e^t - \frac{1}{4}\sin(t^4) + C}$$

i. $\displaystyle\int \frac{(4+\ln(x))^3}{x}\,dx$

$u = 4 + \ln(x)$

$du = \frac{1}{x}\,dx$

$$\int u^3\,du = \frac{u^4}{4} + C$$

$$= \frac{(4+\ln(x))^4}{4} + C$$

j. $\int \frac{x^3+5}{x}\,dx$

$$\int x^2 + \frac{5}{x}\,dx = \boxed{\frac{x^3}{3} + 5\ln(|x|) + C}$$

k. $\int e^{\pi x}\,dx$

$$\int e^{\pi x}\,dx = \int e^u \frac{1}{\pi}\,du = \frac{1}{\pi}e^u + C$$

$$= \boxed{\frac{1}{\pi}e^{\pi x} + C}$$

$u = \pi x$

$du = \pi\,dx$

l. $\int \sec^2(x)\tan^5(x)\,dx$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$$\int u^5\,du = \frac{u^6}{6} = \boxed{\frac{1}{6}\tan^6(x) + C}$$