Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $\frac{dy}{dx} =$, or similar.
- **Draw a box around your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(t) = t\sin(t)$

$$\boxed{f'(t) = t\cos(t) + \sin(t)}$$

b. $f(x) = e^{(7-x^5)}$

$$\boxed{f'(x) = \left(e^{7-x^5}\right)\left(-5x^4\right)}$$

c. $f(x) = \sqrt{3x + \ln(6x)}$ $= \left(3x + \ln(6x)\right)^{1/2}$

$$\boxed{f'(x) = \frac{1}{2}\left(3x + \ln(6x)\right)^{-1/2}\left(3 + \frac{1}{6x}(6)\right)}$$

$$= \frac{1}{2}\left(3x + \ln(6x)\right)^{-1/2}\left(3 + \frac{1}{x}\right)$$

d. $f(x) = \dfrac{\cos(x/2)}{x^6}$ $= \cos\left(\frac{x}{2}\right)\left(x^{-6}\right)$

$$\boxed{f'(x) = \frac{x^6\left(-\sin\left(\frac{x}{2}\right)\left(\frac{1}{2}\right)\right) - \cos\left(\frac{x}{2}\right)\left(6x^5\right)}{\left(x^6\right)^2}}$$

$$f'(x) = \cos\left(\frac{x}{2}\right)\left(-6x^{-7}\right) + x^{-6}\left(-\sin\left(\frac{x}{2}\right)\left(\frac{1}{2}\right)\right)$$

e. $f(x) = \dfrac{1}{9x} + \sqrt{5-x}$ $= (9x)^{-1} + (5-x)^{1/2}$

$$\boxed{f'(x) = -(9x)^{-2}(9) + \frac{1}{2}(5-x)^{-1/2}(-1)}$$

$$= \frac{1}{9}\left(-x^{-2}\right) - \frac{1}{2}(5-x)^{-1/2}$$

$$= -\frac{1}{9x^2} - \frac{1}{2\sqrt{5-x}}$$

f. $f(\theta) = \ln(\sec\theta + \tan\theta)$

$$\boxed{f'(\theta) = \left(\frac{1}{\sec\theta + \tan\theta}\right)\left(\sec\theta\tan\theta + (\sec(\theta))^2\right)}$$

$$= \frac{\sec\theta(\tan\theta + \sec\theta)}{\sec\theta + \tan\theta}$$

$$= \sec\theta$$

g. $f(q) = \dfrac{q\ln(q)}{\ln 2} = \dfrac{1}{\ln(2)}\left(q\ln(q)\right)$

$$\boxed{f'(q) = \frac{1}{\ln(2)}\left(q\cdot\frac{1}{q} + \ln(q)\right)}$$

$$= \frac{1+\ln q}{\ln(2)}$$

h. $f(x) = \dfrac{\cos(x)}{\sin(x)} = \cot(x)$

$$\boxed{f'(x) = -(\csc(x))^2}$$

alternately

$$f'(x) = \frac{\sin(x)(-\sin(x)) - \cos(x)\cos(x)}{(\cos(x))^2}$$

$$= -\frac{1}{(\cos(x))^2} = -(\csc(x))^2$$

alternate 2: $f(x) = \cos(x)\cdot(\sin(x))^{-1}$

$$f'(x) = \cos(x)(-\sin(x))^{-2}(\cos(x)) + (\sin(x))^{-1}(-\sin(x))$$

$$= -\left(\frac{\cos(x)}{\sin(x)}\right)^2 - \frac{\sin(x)}{\sin(x)}$$

$$= -1 - (\cot(x))^2$$

$$= -\csc^2(x).$$

(since $\cos^2(x) + \sin^2(x) = 1$, $\dfrac{\cos^2(x)}{\sin^2(x)} + \dfrac{\sin^2(x)}{\sin^2(x)} = \dfrac{1}{\sin^2(x)} \Rightarrow \cot^2(x) + 1 = \csc^2(x)$)

i. $y = \pi\left(\dfrac{6+x}{12}\right)^5 = \dfrac{\pi}{12^5}(6+x)^5$

$$\boxed{y' = \frac{5\pi}{12^5}(6+x)^4(1)}$$

$$= 5\pi\left(\frac{6+x}{12}\right)^4\left(\frac{1}{12}\right)$$

j. $f(x) = \left(\sin(x^3 + e^3)\right)^5$

$$\boxed{f'(x) = 5\left(\sin(x^3+e^3)\right)^4\left(\cos(x^3+e^3)\right)(3x^2)}$$

k. $f(x) = \arctan(3x)$ (this is the same as writing $f(x) = \tan^{-1}(3x)$)

$$\boxed{f'(x) = \frac{1}{1+(3x)^2}(3)}$$

l. Find $\dfrac{dy}{dx}$ for $2y + x = y\sin(x)$. You must solve for $\dfrac{dy}{dx}$.

$$2\frac{dy}{dx} + 1 = y\cos(x) + \sin(x)\frac{dy}{dx} \Rightarrow$$

$$\left(2 - \sin(x)\right)\frac{dy}{dx} = y\cos(x) - 1 \Rightarrow$$

$$\boxed{\frac{dy}{dx} = \frac{y\cos(x) - 1}{2 - \sin(x)}}$$