Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \sin^{-1}(e^x)$ $= \arcsin(e^x)$

$$f'(x) = \frac{1}{\sqrt{1-(e^x)^2}}(e^x)$$

b. $f(x) = e^{\cos x}$

$$f'(x) = e^{\cos(x)}(-\sin(x))$$

c. $f(x) = \sqrt{3x + \ln(4x^2)}$

$$f'(x) = \frac{1}{2}(3x + \ln(4x^2))^{-1/2}\left(3 + \frac{1}{4x^2}(8x)\right)$$

d. $f(x) = \dfrac{\tan x}{x^3+1}$

$$f'(x) = \frac{(x^3+1)\cdot \sec^2(x) - \tan(x)(3x^2)}{(x^3+1)^2}$$

e. $f(x) = \dfrac{1}{2x} + \dfrac{7x^2}{2}$ $= \frac{1}{2}x^{-1} + \frac{7}{2}x^2$

$$f'(x) = \frac{1}{2}(-x^{-2}) + \frac{7}{2}(2x)$$

f. $f(x) = \dfrac{\cot x}{\csc x}$ $= \dfrac{\cos(x)}{\sin(x)} \cdot \dfrac{1}{\frac{1}{\sin x}} = \dfrac{\cos x}{\sin x} \cdot \sin x = \cos x$

Method 1:

$$f'(x) = \frac{\csc x(-\csc^2 x) - (\cot x)(-\csc x \cot x)}{(\csc x)^2}$$

$$= \frac{-\csc^3 x + \csc x \cot^2 x}{(\csc x)^2} = -\csc x + \frac{\cot^2 x}{\csc x}$$

$$= \frac{-\csc^2 x + \cot^2 x}{\csc x} = \frac{-1}{\csc x} = -\sin x$$

Method 2:

$f(x) = \dfrac{\cot x}{\csc x} = \cos x$

$f'(x) = -\sin(x)$:)

$\dfrac{\sin^2 x}{\sin^2 x} + \dfrac{\cos^2 x}{\sin^2 x} = \dfrac{1}{\sin^2 x}$

$1 + \cot^2 x = \csc^2 x$

$\cot^2 x - \csc^2 x = -1$

g. $f(x) = 4x^6 + 3x^5 - 5x^2 + \sin(\pi/2)$

$f'(x) = 4(6x^5) + 3(5x^4) - 5(2x) + 0$

h. $f(t) = t\ln t + t^2$

$f'(t) = t \cdot \frac{1}{t} + \ln(t)(1) + 2t$

i. $f(x) = x\sin(2-5x)$

$f'(x) = x \cdot \cos(2-5x)(-5) + \sin(2-5x)$

j. $f(x) = \ln\left(\frac{x^2}{e^x}\right)$

$$f'(x) = \frac{1}{\frac{x^2}{e^x}}\left(\frac{e^x(2x) - x^2 e^x}{e^{2x}}\right)$$

k. $f(x) = (5^x - x^5)^2$

$$f'(x) = 2(5^x - x^5)(5^x \ln 5 - 5x^4)$$

l. Find $\frac{dy}{dx}$ for $x^2 + y^2 = \cos(xy)$. You must solve for $\frac{dy}{dx}$.

$$2x + 2y\frac{dy}{dx} = -\sin(xy)\left(x\frac{dy}{dx} + y\right) \Rightarrow$$

$$2x + 2y\frac{dy}{dx} = -x\sin(xy)\frac{dy}{dx} - y\sin(xy) \Rightarrow$$

$$\frac{dy}{dx}(2y + x\sin(xy)) = -2x - y\sin(xy) \Rightarrow$$

$$\frac{dy}{dx} = \frac{-2x - y\sin(xy)}{2y + x\sin(xy)}$$