Name: Key

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- You must show sufficient work to justify your final expression. A correct answer for a nontrivial computation with no supporting work will be marked as incorrect.
- Your final answers **must start with** $f'(x) =$, $\frac{dy}{dx} =$, or similar.
- $\boxed{\textbf{Draw a box around your final answer.}}$

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \dfrac{1}{3x} + \sqrt{3x} = \frac{1}{3}x^{-1} + (3x)^{\frac{1}{2}}$

$$\boxed{f'(x) = -\frac{1}{3}x^{-2} + \frac{1}{2}(3x)^{-\frac{1}{2}} \cdot 3}$$

b. $f(x) = 4x - x^2 + 2\tan(x)$

$$\boxed{f'(x) = 4 - 2x + 2\sec^2 x}$$

c. $y = \arcsin(x^3)$

$$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1-x^6}} \cdot (3x^2)}$$

d. $f(x) = a^{\sin(x)}$ where a is a constant, $a > 1$

$$\boxed{f'(x) = a^{\sin x} \cdot \ln a \cdot \cos x}$$

e. $f(x) = \sqrt{x + \ln(2x)}$

$$\boxed{f'(x) = \frac{1}{2}\left(x + \ln(2x)\right)^{-\frac{1}{2}} \cdot \left(1 + \frac{1}{2x} \cdot 2\right)}$$

f. $f(x) = \sec\left(\frac{x}{x+1}\right)$

$$\boxed{f'(x) = \sec\left(\frac{x}{x+1}\right)\tan\left(\frac{x}{x+1}\right) \cdot \left(\frac{x+1-x}{(x+1)^2}\right)}$$

g. $f(x)=\sqrt{1+x^2}$

$$f'(x)=\frac{1}{2}(1+x^2)^{-\frac{1}{2}}\cdot(2x)$$

h. $f(x)=\dfrac{e^x}{\sqrt{x}}$

$$\boxed{f'(x)=\frac{e^x\sqrt{x}-e^x\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{(\sqrt{x})^2}}$$

i. $f(x)=\left(\ln(x^2+e^2)\right)^5$

$$\boxed{f'(x)=5\left(\mathrm{Ln}(x^2+e^2)\right)^4\cdot\frac{1}{x^2+e^2}\cdot(2x)}$$

j. $f(x) = \dfrac{x\ln(x)}{2}$ $= \frac{1}{2}x\ln x$

$$\boxed{f'(x) = \frac{1}{2}\ln x + \frac{1}{2}x\left(\frac{1}{x}\right)}$$

k. $f(x) = e^{\pi x+1} + \sqrt{3}\cot(\pi x)$

$$\boxed{f'(x) = e^{\pi x+1}\cdot(\pi) + \sqrt{3}\left(-\csc^2(\pi x)\cdot\pi\right)}$$

l. Find $\dfrac{dy}{dx}$ for $2x + y = \cos(xy)$. You must solve for $\dfrac{dy}{dx}$.

$$\frac{d}{dx}\left[2x+y\right] = \frac{d}{dx}\left[\cos(xy)\right]$$

$$2 + \frac{dy}{dx} = -\sin(xy)\left(y + x\frac{dy}{dx}\right)$$

$$2 + \frac{dy}{dx} = -\sin(xy)y - \sin(xy)x\frac{dy}{dx}$$

$$2 + \sin(xy)y = -\frac{dy}{dx} - \sin(xy)x\frac{dy}{dx}$$

$$2 + \sin(xy)\cdot y = \frac{dy}{dx}\left(-1 - \sin(xy)\cdot x\right)$$

$$\boxed{\frac{2 + y\sin(xy)}{-1 - x\sin(xy)} = \frac{dy}{dx}}$$