Name: Key

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- You must show sufficient work to justify your final expression. A correct answer for a nontrivial computation with no supporting work will be marked as incorrect.
- Your final answers **must start with** $f'(x) =$, $\frac{dy}{dx} =$, or similar.
- **Draw a box around your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $g(x) = e^{3x}\tan(x)$

$g'(x) = \boxed{3e^{3x}\tan x + e^{3x}\sec^2 x}$

b. $h(x) = \csc(x^3)$

$h'(x) = \boxed{-\csc(x^3)\cot(x^3)\cdot 3x^2}$

c. $f(x) = \dfrac{5}{3x} + \dfrac{x^2}{\sqrt{5}} - \dfrac{\pi^2}{3}$

$f'(x) = \boxed{-\frac{5}{3}x^{-2} + \frac{2x}{\sqrt{5}}}$

d. $f(x) = x\arcsin(x)$

$f'(x) = \boxed{\arcsin x + \dfrac{x}{\sqrt{1-x^2}}}$

e. $y = (x^{0.4}+4)^{-1/5}$

$\dfrac{dy}{dx} = \boxed{-\frac{1}{5}(x^{0.4}+4)^{-6/5} \cdot 0.4x^{-0.6}}$

f. $f(t) = \sqrt{t^2+\cos^2(t)}$

$f'(t) = \boxed{\frac{1}{2}(t^2+\cos^2(t))^{-\frac{1}{2}}}$

g. $g(x) = \ln(8 + \sin(x^4))$

$$g'(x) = \boxed{\frac{1}{8 + \sin(x^4)} \left(\cos(x^4) \cdot 4x^3\right)}$$

h. $f(x) = \dfrac{\sin(\pi x)}{x^3 + x}$

$$f'(x) = \boxed{\frac{\cos(\pi x) \cdot \pi \, (x^3 + x) - \sin(\pi x)\,(3x^2 + 1)}{(x^3 + x)^2}}$$

i. $y = e^{(x^2)} + \sec(5x)$

$$\frac{dy}{dx} = \boxed{e^{x^2} \cdot 2x + \sec(5x)\tan(5x) \cdot 5}$$

j. $f(x) = \sqrt{2}\cos(1+e^{-Nx})$ (Assume N is a fixed positive constant.)

$f'(x) = \boxed{-\sqrt{2}\sin(1+e^{-Nx})\cdot(e^{-Nx}\cdot(-N))}$

k. $j(x) = \dfrac{x\ln(x) - \sqrt{x}}{x^2} = x^{-1}\ln x - x^{-3/2}$

$j'(x) = \boxed{-x^{-2}\ln x + x^{-1}\cdot x^{-1} + \frac{3}{2}x^{-5/2}}$

l. Find $\dfrac{dy}{dx}$ for $1 + xy = x^3 + y^2$

$\frac{d}{dx}[1 + xy] = \frac{d}{dx}[x^3 + y^2]$

$y + x\frac{dy}{dx} = 3x^2 + 2y\frac{dy}{dx}$

$y - 3x^2 = -x\frac{dy}{dx} + 2y\frac{dy}{dx}$

$y - 3x^2 = (-x+2y)\frac{dy}{dx}$

$\boxed{\frac{dy}{dx} = \frac{y - 3x^2}{-x + 2y}}$