Name: Key

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- You must show sufficient work to justify your final expression. A correct answer for a nontrivial computation with no supporting work will be marked as incorrect.
- Your final answers **must start with** $f'(x) =$, $\frac{dy}{dx} =$, or similar.
- **Draw a box around your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(t) = e^t(3 - t^4)$

$f'(t) = \boxed{e^t(3-t^4) + e^t(-4t^3)}$

b. $r(\theta) = \tan\left(\sqrt{3} + \theta^2\right)$

$r'(\theta) = \boxed{\sec^2(\sqrt{3} + \theta^2) \cdot 2\theta}$

c. $g(z) = (3z - 4)(z^2 + 7)$

$g'(z) = \boxed{3(z^2+7) + (3z-4)(2z)}$

d. $f(x) = 3\cos(x) + x\sqrt{x+1}$

$$f'(x) = \boxed{-3\sin x + \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}}$$

e. $f(r) = \dfrac{r^3 + \sqrt{r} - 2}{r}$ $= r^2 + r^{-\frac{1}{2}} - 2r^{-1}$

$$f'(r) = \boxed{2r - \frac{1}{2}r^{-\frac{3}{2}} + 2r^{-2}}$$

f. $G(x) = \left(\dfrac{x - \ln(4)}{2}\right)^3$

$$G'(x) = 3\left(\frac{x - \ln(4)}{2}\right)^2 \cdot \frac{1}{2}$$

g. $f(y) = e + \cos(y^{\pi})$

$$f'(y) = \boxed{-\sin(y^{\pi}) \cdot (\pi y^{\pi-1})}$$

h. $f(x) = \dfrac{2\sec(bx)}{3x^3}$ (where b is a constant)

$$f'(x) = \boxed{\frac{2b\sec(bx)\tan(bx)(3x^3) - 2\sec(bx)(9x^2)}{(3x^3)^2}}$$

i. $y = x^{1/4}e^{-\sin(x)}$

$$\frac{dy}{dx} = \frac{1}{4}x^{-3/4}e^{-\sin x} + x^{1/4}e^{-\sin x}(-\cos x)$$

j. $y(t) = \ln(2t + \sin(t^2))$

$$y'(t) = \boxed{\frac{1}{2t + \sin(t^2)}\left(2 + \cos(t^2)(2t)\right)}$$

Cos

k. $g(x) = \arctan(e^x)$

$$g'(x) = \boxed{\frac{1}{1 + (e^x)^2} \cdot e^x}$$

l. Compute $\frac{dy}{dx}$ if $\ln y - 5x = x^2 y$. You $\boxed{\text{must}}$ solve for $\frac{dy}{dx}$.

$$\frac{d}{dx}\left[\ln y - 5x\right] = \frac{d}{dx}\left[x^2 y\right]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} - 5 = 2xy + x^2 \frac{dy}{dx}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} - x^2 \frac{dy}{dx} = 2xy + 5$$

$$\left(\frac{1}{y} - x^2\right) \frac{dy}{dx} = 2xy + 5$$

$$\frac{dy}{dx} = \boxed{\frac{2xy + 5}{\frac{1}{y} - x^2}}$$