Name: Key

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. **[12 points]** Compute the derivatives of the following functions.

   a. $f(x) = \dfrac{x^3 - 4x^2}{\sec(2x)}$

   $f'(x) = \dfrac{(3x^2 - 8x)\sec(2x) - (x^3 - 4x^2)\sec(2x)\tan(2x)\cdot 2}{\sec^2(2x)}$

   b. $g(x) = x^{-2}(x^3 + 2x - x^{-\frac{1}{2}})$

   $g'(x) = -2x^{-3}\left(x^3 + 2x - x^{\frac{1}{2}}\right) + x^{-2}\left(3x^2 + 2 + \frac{1}{2}x^{-\frac{3}{2}}\right)$

   c. $h(x) = \ln\left(6 + 6x^{0.6}\right)$

   $h'(x) = \dfrac{1}{6 + 6x^{0.6}}\left(3.6x^{-0.4}\right)$

d. $r(\theta) = \csc\left(\frac{9}{\theta}\right)$

$r'(\theta) = -\csc\left(\frac{9}{\theta}\right)\cot\left(\frac{9}{\theta}\right)\cdot(-9\theta^{-2})$

e. $f(x) = \arctan(2-x)$

$f'(x) = \frac{1}{1+(2-x)^2}(-1)$

f. $p(t) = \sqrt{16t^3}$

$p'(t) = \frac{1}{2}(16t^3)^{-\frac{1}{2}}\cdot(48t^2)$

g. $f(x) = x^a + b^x$ where a and b are fixed constants

$f'(x) = a x^{a-1} + b^x \ln(b)$

h. $g(x) = \sin(e^x + \cos(2x))$

$g'(x) = \cos(e^x + \cos(2x)) \cdot (e^x - \sin(2x) \cdot 2)$

i. $y = \dfrac{\ln x}{3+6x}$

$\dfrac{dy}{dx} = \dfrac{\frac{1}{x}(3+6x) - \ln x\,(6)}{(3+6x)^2}$

j. $f(x) = \tan\left(\dfrac{3x}{2-x^2}\right)$

$$f'(x) = \sec^2\left(\frac{3x}{2-x^2}\right) \cdot \left(\frac{3(2-x^2) - 3x(-2x)}{(2-x^2)^2}\right)$$

k. $y = \log_2(x) + \log_2(8)$

$$\frac{dy}{dx} = \frac{1}{x \ln 2} + 0$$

l. Find $\dfrac{dy}{dx}$ for $x^2e^y = x + y$

$$\frac{d}{dx}\left[x^2e^y\right] = \frac{d}{dx}\left[x + y\right]$$

$$2xe^y + x^2e^y \cdot \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$2xe^y - 1 = \frac{dy}{dx} - x^2e^y \cdot \frac{dy}{dx}$$

$$2xe^y - 1 = \left(1 - x^2e^y\right) \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2xe^y - 1}{1 - x^2e^y}$$